T-61.3040 Statistical Signal Modeling
Exam 15.12.2014

In the exam, you are allowed to have a calculator (non-programmable or memory emptied) and basic mathematical tables (no tables containing material directly associated with the course). For example, the book "Beta Mathematics Handbook for Science and Engineering" by Rade and Westergreen contains material too directly associated with the course, so you are NOT allowed this book at the exam.

The results of the exam will be announced through the Noppa system. Please answer in English.

1. (max 10p) Explain the following topics briefly but covering the most important properties:

a) Wide-Sense Stationarity (WSS) (1p)

b) Ergodicity in the mean (2p)

c) Wiener-Khinchin Theorem (2p)

d) The Minimum Variance Method (3p)

e) ARCH model (2p)

2. (max 10p) Given the following values of a real-valued process $x(n)$: $x(0) = 1$, $x(1) = 2$, $x(2) = 2$:

a) Estimate an autocorrelation matrix of size 3×3 so that the result is positive semi-definite. Show that this result is positive semi-definite. (3p)

b) Model the process $x(n)$ as an AR(2) process. (3p)

c) What are the variance $\mathrm{Var}(x(n))$ and conditional variance $\mathrm{Var}(x(3)|x(2), x(1), x(0))$ of the modeled process? (4p)

3. (max 10p) Multiple choices questions. The following five questions have different proposed answers. Several can be correct. You have to give your answer along with your confidence ("High" or "Low") for each answer. Grading for each of these questions is then:

- +2 if the answer is correct and confidence High
- +1 if the answer is correct and confidence Low
- 0 if the answer is missing
- −1 if the answer is wrong and confidence Low
- −2 if the answer is wrong and confidence High

Write on your answer sheet the correct answer A, B, C, D,...) along with the confidence you have (High or Low) for that question. For example, "A, Low" is a proper way of answering a question. No need to justify your answers. Total score for this question is between 0 and 10 (you cannot get a negative score).

1) A *regular* process can be obtained, in general, by filtering:

A) White noise of fixed variance σ_0^2 by a LSI filter

B) Any noise of fixed variance σ_0^2 by a causal and stable filter

C) Any noise of fixed variance σ_0^2 by a LSI filter

D) White noise of fixed variance σ_0^2 by a causal and stable filter

E) None of the previous answers is correct

2) The Pisarenko method is used to estimate the power spectrum of a WSS process $x(n)$. The pseudo-spectrum is calculated and:

A) The pseudo-spectrum is a very good estimation of the power spectrum

B) In the pseudo-spectrum, some peaks appear for the main frequencies of the process $x(n)$

C) In the pseudo-spectrum, some peaks appear for the main frequency of the process $x(n)$

D) The pseudo-spectrum is infinitely small for the main frequencies of the process $x(n)$

E) The pseudo-spectrum is infinite for the main frequencies of the process $x(n)$

F) None of the previous answers is correct

3) Which of the following methods has the best resolution (you can assume than N is large and that the following methods parameters L, K, M have values around $\sqrt{N}$)?

A) Blackmann-Tukey's method

B) Periodogram method

C) Welch's method

D) Bartlett's method

4) The autocorrelation sequence $r_y(k)$ of the output $y(n)$ of a stable LSI filter with unit sample response $h(n)$ filtering a WSS process with autocorrelation $r_x(k)$ can be calculated as:

A) $r_y(k) = r_x(k) \times h(k)$

B) $r_y(k) = r_x(k) * r_{xy}(k)$

C) $r_y(k) = (r_x(k) * h(k)) \times h^*(-k)$

D) $r_y(k) = r_x^*(-k) \times (h(k) * h^*(-k))$

E) None of the previous answers is correct

5) For jointly WSS processes, the LMS algorithm converges in the mean if the independence assumption is satisfied and the step size μ satisfies (with $\lambda_{\max}, \ldots, \lambda_{\min}$ the ordered eigenvalues of the autocorrelation matrix $\mathbf{R}_x$):

A) $0 < \mu < \frac{2}{\lambda_{\max}}$

B) $\mu < 0$

C) Minimum of the power spectrum $< \mu <$ Maximum of the power spectrum

D) $0 < \mu < \frac{2}{\lambda_{\min}}$

E) $\mu = \sum_{k=0}^{p} \lambda_k$

F) None of the previous answers is correct

4. (max 10p) The process $x(n) = A\exp(jn\omega) + v(n)$ consists of one complex sinusoid in white noise. If the autocorrelation matrix is

$$\mathbf{R}_x = \begin{bmatrix} 3 & 2(1-j) \\ 2(1+j) & 3 \end{bmatrix}$$

then:

a) What is the variance of the noise? (3p)

b) What is the power $P = |A|^2$? (3p)

c) What is the frequency ω of the signal? (4p)