Rak-54.3110 Plate and Shell Structures, Fall 2014, Exam for Part II
Thursday, 18.12.2014, 13:00-15:00

Closed book examination. Write your name, study program and the course code on each answering sheet.

1. Which of the following statements are true and which are false? If the statement is false, give an explanation why.

   a) If the Gaussian curvature of a shell middle surface vanishes at every point, then the shell is actually a flat plate/membrane.

   b) Shell membrane theory may also be used to analyse sails and cloths made of fabric if tensile stresses are obtained at all sections.

   c) In the linear theory of thin elastic shells, the displacements of any point of a shell are assumed to be small in comparison to its thickness.

   d) When designing a thin shell it is important to prevent inextensional deformation.

   e) In the shell membrane theory, the transverse deflection can always be constrained on the edges without disturbing the membrane state of stress.

   f) If purely inextensional displacements with vanishing membrane strains have been prevented by the shell design, then bending effects are usually confined to relatively small areas near edges and other disturbances.

   g) A finite element program has only one shell element that includes out-of-plane/transverse shear deformations. Such a program is not suitable for analysing very thin shell structures.

   h) Thin shell structures often buckle at loads much smaller than those predicted by linear stability analysis. This occurs because the governing differential equation cannot be solved properly.

2. The total potential energy of the Reissner-Naghdi model for shallow shells may be written in the form

$$
\begin{aligned}
\mathcal{F}(u_1, u_2, w, \theta_1, \theta_2) = & \frac{1}{2}\int_\omega (N_{11}\beta_{11} + N_{22}\beta_{22} + 2S\beta_{12})\, dxdy \\
& + \frac{1}{2}\int_\omega (Q_1\gamma_1 + Q_2\gamma_2)\, dxdy \\
& + \frac{1}{2}\int_\omega (M_{11}\kappa_{11} + M_{22}\kappa_{22} + 2H\kappa_{12})\, dxdy \\
& - \int_\omega (p_1u_1 + p_2u_2 + p_nw)\, dxdy,
\end{aligned}
$$

   where

$$
\begin{aligned}
&N_{11} = D_m(\beta_{11} + \nu\beta_{22}), \quad N_{22} = D_m(\nu\beta_{11} + \beta_{22}), \quad S = D_m(1-\nu)\beta_{12}, \\
&Q_1 = Gt\gamma_1, \quad Q_2 = Gt\gamma_2, \\
&M_{11} = D(\kappa_{11} + \nu\kappa_{22}), \quad M_{22} = D(\nu\kappa_{11} + \kappa_{22}), \quad H = D(1-\nu)\kappa_{12},
\end{aligned}
$$

and where further

$$\beta_{11} = \frac{\partial u_1}{\partial x} - \frac{w}{R_1}, \quad \beta_{22} = \frac{\partial u_2}{\partial y} - \frac{w}{R_2}, \quad \beta_{12} = \frac{1}{2}\left(\frac{\partial u_1}{\partial y} + \frac{\partial u_2}{\partial x}\right),$$

$$\gamma_1 = -\theta_1 + \frac{\partial w}{\partial x}, \quad \gamma_2 = -\theta_2 + \frac{\partial w}{\partial y},$$

$$\kappa_{11} = -\frac{\partial \theta_1}{\partial x}, \quad \kappa_{22} = -\frac{\partial \theta_2}{\partial y}, \quad \kappa_{12} = -\frac{1}{2}\left(\frac{\partial \theta_1}{\partial y} + \frac{\partial \theta_2}{\partial x}\right),$$

and

$$D_m = \frac{Et}{1-\nu^2}, \quad D = \frac{Et^3}{12(1-\nu^2)}, \quad G = \frac{E}{2(1+\nu)}.$$

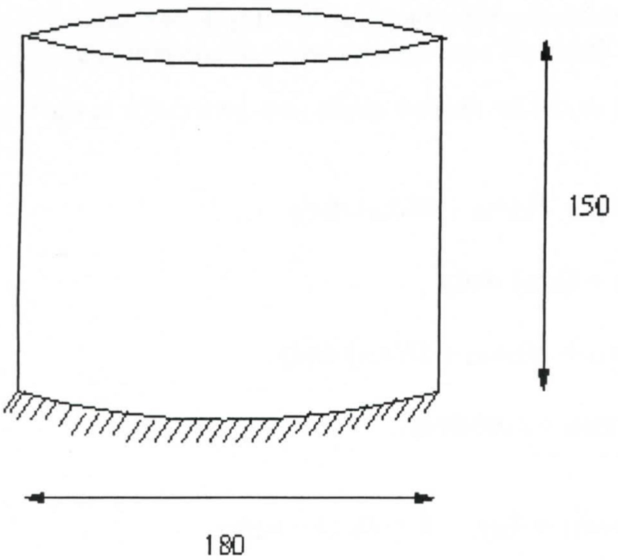
(a) What is the physical interpretation of the quantities

$$u_1, u_2, w, \theta_1, \theta_2, N_{11}, N_{22}, S, Q_1, Q_2, M_{11}, M_{22}, H, p_1, p_2, p_n, R_1, R_2, t, \nu$$

Give also the units of the quantities in the SI system.

(b) Describe how the energy functional of the corresponding Kirchhoff-Love type model is obtained formally from the energy functional of the Reissner-Naghdi model by neglecting transverse shear deformations. What form do the strain-displacement relations take in the Kirchhoff-Love type model?

3. Utilize the formula $n_{cr} \approx 0.6Et^2/R$ for the critical membrane force to estimate whether the circular cylindrical container shell of Figure 1 will buckle due to self-weight.



Height 150 m
Bottom diameter 180 m
Wall thickness 0.10 m
Density 2400 kg/m^3
Young's modulus 30 GPa
Acceleration due to gravity 10 m/s^2
Knock-down factor 1/6

Figure 1: Cylindrical container shell.