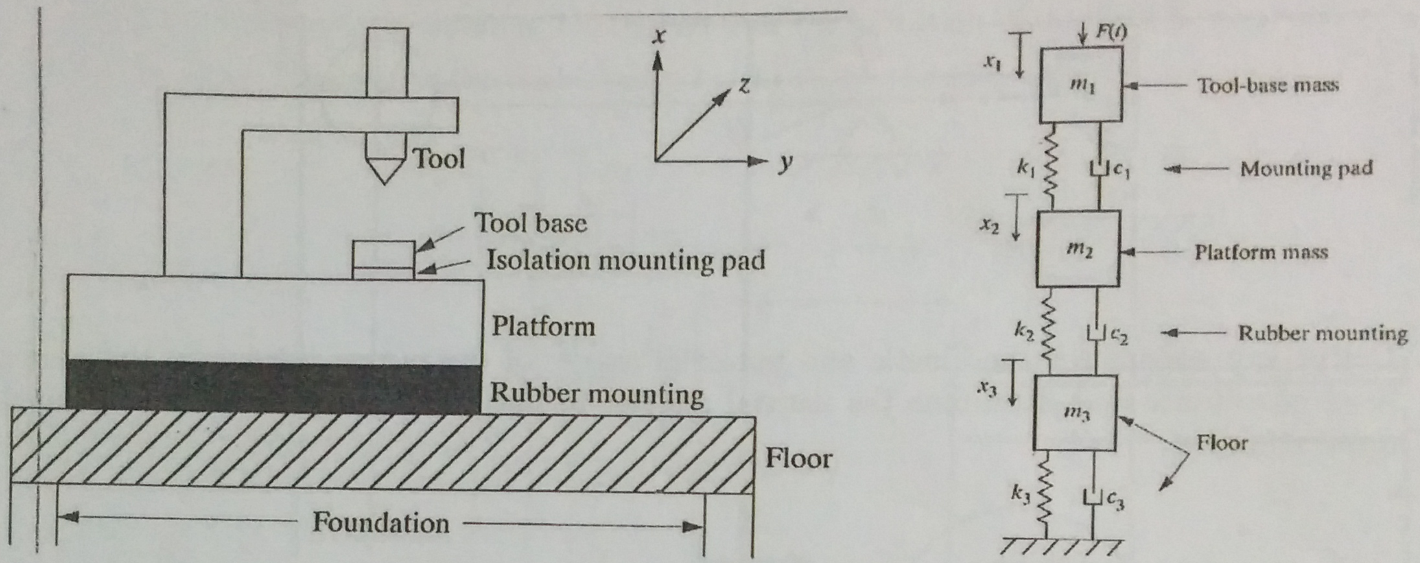
Mid-term exam 2/Kul-49.3400/2014 Dynamics of structures 18.12.2014

**Choose either the mid-term or the final exam. Do not do both. Turn the page for the final exam. Tasks 3 and 4 are the same in both exams.**

1. The hole punching machine shown on the left can be modeled as shown on the right. Write the equations of motion for the machine according to the model on the right. Present the equations of motion in matrix form.

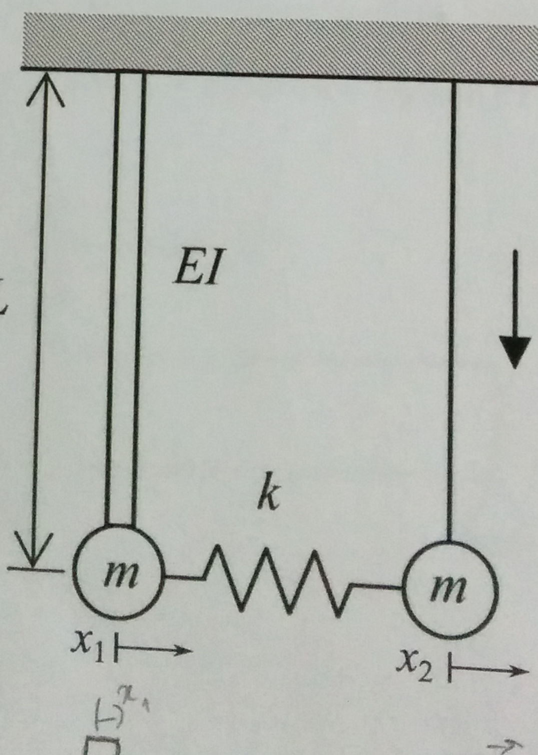



2. The beam and the pendulum connected by a spring form a vibrational system as shown in the figure. Calculate the angular eigenfrequencies using $\det(K - \omega^2 M) = 0$, and the mode shapes $\mathbf{u}_i$ of the system related to horizontal movement. The masses of the beam and pendulum are negligible compared to masses $m$. Use values $EI = mgL^2$, $k = 1.5mg/L$ and $k_{pend} = mg/L$. Horizontal deflection of the tip of the beam: $v = \frac{FL^3}{3EI}$.

$3x_1 = x_2 \Leftrightarrow x_1 = \frac{x_2}{3}$

$\dot{x}_1 = \frac{\dot{x}_2}{3}$

$4{,}5\,\frac{mg}{L}$




$-L \begin{vmatrix} -\theta \\ 1 \end{vmatrix} \wedge \begin{vmatrix} k(x_1 - x_2) \\ 0 \end{vmatrix}$

$= + Lk(x_1 - x_2)$

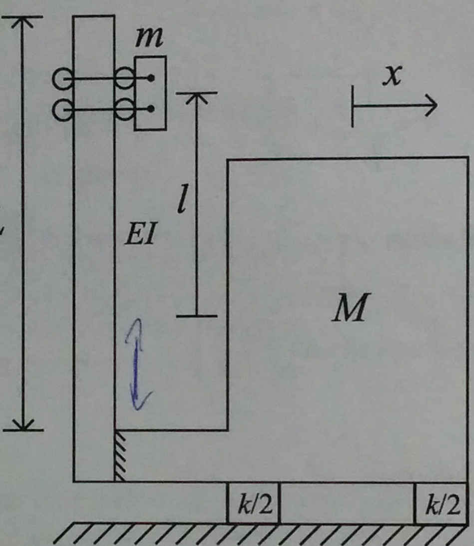
$-k(x_1 - x_2) - k_{pend}$

$k(x_1 - x_2)$

3. It is desired that a machine, which has a mass $M = 1200$ kg, could be used with angular rotation frequencies $\omega = 60 \ldots 200$ rad/s. Due to strong vibrations during operation, a steel beam is attached to the machine ($EI = 400$ Nm$^2$ and $L = 0.5$ m); The location $l$ of the mass $m = 4$ kg attached to the beam, can be controlled continuously between $l = L/3 \ldots L$ (free end of the beam). The vibration of the machine in $x$-direction (vertical direction is neglected) is caused by a mass eccentricity, so the control parameter for the system is the rotation frequency $\omega$. It can be assumed that damping, and the mass of the beam are small. The deflection of a cantilever beam of length $L$ in the free end is $v = \frac{FL^3}{3EI}$.

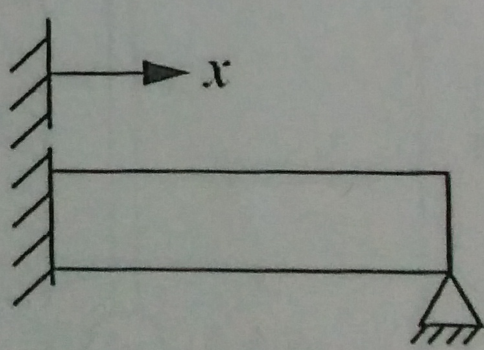
a) Determine a control equation $l = l(\omega)$ so that the vibration of mass $M$ is minimal.

b) For which values of $m$ the control zone $l = L/3 \ldots L$ is sufficient?




4. Determine the characteristic equation of bending vibrations of a Euler-Bernoulli beam, which is fixed from one end ($x = 0$, *fixed*) and pinned from the other end ($x = L$, *pinned*). Characteristic equation is the equation, which is needed to calculate the angular eigenfrequencies of the beam. The shape function of the separated trial function of the beam is

$$X(x) = C_1 \cos \beta x + C_2 \sin \beta x + C_3 \cosh \beta x + C_4 \sinh \beta x \, .$$




$X(0) = 0$
$X'(0) = 0$
$X(l) = 0$
$X'(l) = 0$