Aalto University School of Science, Department of Information and Computer Science TJ

T-79.5104 Advanced Course in Computational Logic

Examination, December 10, 2014

Assignment 1 (Max 10p)

(a) Define what it means that (i) a frame logic **L** is decidable and (ii) **L** is semi-decidable. (4p)

(b) Given a frame logic **L**, define the *logical consequence* relation $\Sigma \models_{\mathbf{L}} \Upsilon \Longrightarrow P$. (3p)

(c) Describe in which sense the relation above is *monotonic*. (3p)

Assignment 2 (Max 10p)

(a) Define the Hilbert-style proof system for modal logic **K**.

(b) Derive the following inference rule using the rules of this system:

$$\frac{\Box(P \wedge Q)}{\Box P \wedge \Box Q}$$

Assignment 3 (Max 10p) Use the *tableau method* to determine whether the following claims hold. Give a counter-model based on the tableau if appropriate. (Symbols P and Q denote atomic propositions below.)

(a) The sentence $(\Diamond P \wedge \Diamond Q) \leftrightarrow \Diamond(P \wedge Q)$ is **K**-valid.

(b) The sentence $\Box(\Box Q \rightarrow \Box P)$ is a **KB**-consequence (in the logic based on symmetric frames) of the *global* premise $\Box Q \rightarrow Q$ and the *local* premise $\Box\Box(\Diamond\neg P \rightarrow \neg Q)$.

Assignment 4 (Max 10p) Show that the following two modal logics coincide: **KB4** based on symmetric and transitive frames and **KB5** based on symmetric and Euclidean frames.

Assignment 5 (Max 10p)

(a) Define the following concepts in $\mathcal{ALC}$ extended by *inverse roles* using the concept name `Worker` and the role name `supervises`:

1. A manager (a worker who supervises at least one worker)
2. A director general (a manager who supervises only mangers and is not supervised by any worker)

(b) Consider a knowledge base $(\mathcal{T}, \mathcal{A})$ having TBox $\mathcal{T} = \{A \sqsubseteq C, B \sqsubseteq C\}$ and ABox $\mathcal{A} = \{a : (\exists r.A \sqcup \exists r.B)\}$ where A, B, and C are concept names, r is a role name, and a is an individual name.

Use the tableau algorithm for $\mathcal{ALC}$ to study whether the KB $(\mathcal{T}, \mathcal{A})$ entails that the individual a is an instance of the concept $(\exists r.C)$ and give a counter model if appropriate.

Properties of a relation R:				
	Reflexive:	$\forall s(sRs)$	Serial:	$\forall s \exists t(sRt)$
	Symmetric:	$\forall s \forall t(sRt \rightarrow tRs)$	Euclidean:	$\forall s \forall t \forall u(sRt \wedge sRu \rightarrow tRu)$
	Transitive:	$\forall s \forall t \forall u(sRt \wedge tRu \rightarrow sRu)$		

The name of the course, the course code, the date, your name, your student identifier, and your signature must appear on every sheet of your answers.