

First mid-term exam (19.11.2013, 5pm–7pm)

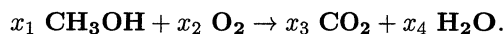
Please fill in the required information onto each answer sheet.

Calculators and mathematical tables are not allowed.

About grading: Every exam problem will be graded from 0 to 6 points. Harmless small errors do not prevent from getting maximal points. You will get points if your answer contains at least some information (relevant definitions, pictures, calculations etc) — empty answer is surely worth zero.

1. Let $u, v, x \in \mathbb{R}^3$, where
$$\begin{cases} u = (6/7, 2/7, -3/7), \\ v = (2/7, 3/7, 6/7), \\ x = (19, 11, 2013). \end{cases}$$

- a) Find norm $\|u\|$.
 - b) Find cross product $w = u \times v$.
 - c) Find $(x \cdot u)u + (x \cdot v)v + (x \cdot w)w$, where $w = u \times v$.
2. Find the smallest positive integers x_1, x_2, x_3, x_4 in the methanol burning reaction formula



Solve this problem in matrix form by using Gauss' elimination process.

3. Let $A = P D P^{-1}$, where

$$P = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 1 \\ 2 & -5 & -3 \end{bmatrix} \quad \text{ja} \quad D = \begin{bmatrix} 8 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 7 \end{bmatrix}.$$

Find all solutions $\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$ to equation $A\mathbf{x} = 8\mathbf{x}$.

(Remark: This can be solved quickly without finding matrices P^{-1} and A . You may compute these matrices, if you don't find other solution method.)