MS-C1420 Fourier analysis (Aalto University)
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**Final examination (12.11.2013, 4pm–8pm)**

Please fill in the required information onto each answer sheet.

**Calculators and mathematical tables are not allowed.**

About grading: Every exam problem will be graded from 0 to 6 points. Harmless small errors do not prevent from getting maximal points. You will get points if your answer contains at least some information (relevant definitions, pictures, calculations etc) — empty answer is surely worth zero.

1. Find the Fourier transform $\widehat{s} : \mathbb{R} \to \mathbb{C}$ of signal $s : \mathbb{R} \to \mathbb{C}$, where
$$s(t) := \begin{cases} 1, \text{ kun } t \in [-1/2, +1/2], \\ 0 \text{ muutoin.} \end{cases}$$
Due to the time symmetry, $\widehat{s}$ in this problem is real-valued, so present your solution accordingly!

2. Let us define transform $As : \mathbb{R} \to \mathbb{C}$ of signal $s : \mathbb{R} \to \mathbb{C}$ by formula
$$As(t) = \int_{\mathbb{R}} K(t,u)\ s(u)\ \mathrm{d}u,$$
where $K : \mathbb{R} \times \mathbb{R} \to \mathbb{C}$ is so-called kernel of $A$ (here we do not study conditions under which this integral makes sense). We say that $A$ is "time invariant", if $As(t-t_0) = As_{t_0}(t)$, where $s_{t_0}(t) = s(t-t_0)$. Show that then
$$K(t,u) = r(t-u)$$
for some $r : \mathbb{R} \to \mathbb{C}$ (that is, then $As = r * s$).

3. Find the discrete-time Fourier transform $\widehat{s} : \mathbb{R}/\mathbb{Z} \to \mathbb{C}$ of digital signal $s : \mathbb{Z} \to \mathbb{C}$, when
$$s(t) = 2^{-|t|}.$$
(Due to the symmetry, the transform here is real-valued, so simplify your answer accordingly!)

4. Find the discrete Fourier transform of signal $s : \mathbb{Z}/4\mathbb{Z} \to \mathbb{C}$, when $s(t) = \mathrm{i}^t$.

5. The Wigner time-frequency distribution $Ws : \mathbb{R} \times \mathbb{R} \to \mathbb{C}$ of signal $s : \mathbb{R} \to \mathbb{C}$ is defined by
$$Ws(t,\nu) := \int_{\mathbb{R}} \mathrm{e}^{-\mathrm{i}2\pi u \cdot \nu}\ s(t+u/2)\ \overline{s(t-u/2)}\ \mathrm{d}u.$$
Find $Ws$, when $s(t) = \mathrm{e}^{-\pi t^2}$.
(Here you may use information $\widehat{s}(\nu) = s(\nu)$, when $s(t) = \mathrm{e}^{-\pi t^2}$.)