MS-C1420 Fourier analysis (Aalto University)
Turunen / Saari

Second mid-term exam (12.11.2013, 4pm–8pm)

Please fill in the required information onto each answer sheet.

Calculators and mathematical tables are not allowed.

About grading: Every exam problem will be graded from 0 to 6 points. Harmless small errors do not prevent from getting maximal points. You will get points if your answer contains at least some information (relevant definitions, pictures, calculations etc) — empty answer is surely worth zero.

1. Let us study periodization $\mathcal{P}s : \mathbb{R}/\mathbb{Z} \to \mathbb{C}$ of an analog non-periodic signal $s : \mathbb{R} \to \mathbb{C}$, where
$$\mathcal{P}s(t) := \sum_{k\in\mathbb{Z}} s(t-k).$$
Show by calculating that $\widehat{\mathcal{P}s}(\nu) = \widehat{s}(\nu)$ for all $\nu \in \mathbb{Z}$.
(Remember to justify your reasoning!)

2. Find the discrete-time Fourier transform $\widehat{s} : \mathbb{R}/\mathbb{Z} \to \mathbb{C}$ of digital signal $s : \mathbb{Z} \to \mathbb{C}$, when
$$s(t) = 2^{-|t|}.$$
(Due to the symmetry, the transform here is real-valued, so simplify your answer accordingly!)

3. Find the discrete Fourier transform of signal $s : \mathbb{Z}/4\mathbb{Z} \to \mathbb{C}$, when $s(t) = \mathrm{i}^t$.

4. The Wigner time-frequency distribution $Ws : \mathbb{R} \times \mathbb{R} \to \mathbb{C}$ of signal $s : \mathbb{R} \to \mathbb{C}$ is defined by
$$Ws(t,\nu) := \int_{\mathbb{R}} \mathrm{e}^{-\mathrm{i}2\pi u\cdot\nu}\; s(t+u/2)\,\overline{s(t-u/2)}\,\mathrm{d}u.$$
Find Ws, when $s(t) = \mathrm{e}^{-\pi t^2}$.
(Here you may use information $\widehat{s}(\nu) = s(\nu)$, when $s(t) = \mathrm{e}^{-\pi t^2}$.)