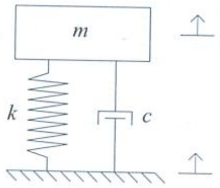
Mid-term exam 1/Kul-49.3400/2014 **Dynamics of Structures** 21.10.2014

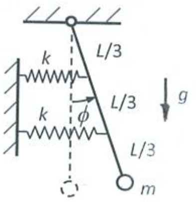
1. a) An accelerometer shows that a structure vibrates back and forth harmonically 82 times in one second. The maximum acceleration is measured to be 50 g ($g = 9.81$ m/s^2). Determine the amplitude of the vibration.

b) The amplitude and natural angular frequency of an undamped harmonic oscillator are measured to be 1 mm and 5 rad/s, respectively. At the instance $t = 0$ the phase shift is measured to be 2 rad. Determine the initial conditions, $x_0 = x(0)$ and $v_0 = v(0)$, of the vibration.

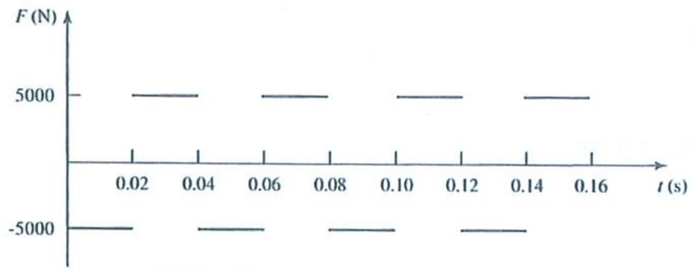
2. A machine having a mass of $m = 350$ kg is attached to a base using a spring k and a damper $c = 1800$ Ns/m. The base is vibrating harmonically in vertical direction with amplitude 0.5 mm and frequency 20 Hz. Determine k so that the displacement of the machine is smaller than 0.1 mm.



3. A mass m is supported by a massless rigid rod (length L) as shown in the figure. Determine the equation of motion of the system and the expression for the natural angular frequency. Use the small angle approximation. The springs attached to the rod are at rest (no extension) when $\phi = 0$. The moment of inertia: $J = mL^2$.



4. Determine the Fourier series representation for the periodic excitation shown in the figure below. $\int \sin ax\, dx = -\frac{1}{a}\cos ax$ and $\int \cos ax\, dx = \frac{1}{a}\sin ax$



$$F(t) = \frac{a_0}{2} + \sum_{i=1}^{\infty} (a_i \cos \omega_i t + b_i \sin \omega_i t) \qquad \omega_i = \frac{2\pi i}{T} \qquad i = 1, 2, \ldots.$$

$$a_0 = \frac{2}{T}\int_0^T F(t)\, dt \qquad a_i = \frac{2}{T}\int_0^T F(t) \cos \omega_i t\, dt \qquad b_i = \frac{2}{T}\int_0^T F(t) \sin \omega_i t\, dt$$

Formulary

Kul-49.3400 Dynamics of machines and structures, autumn 2014

Basic concepts

$$\omega_n = \sqrt{\frac{k}{m}} \qquad \omega_n = 2\pi f_n \qquad \zeta = \frac{c}{2m\omega_n} \qquad \omega_d = \omega_n\sqrt{1-\zeta^2}$$

Harmonic oscillator

$$x(t) = A\sin(\omega_n t + \phi) \qquad A = \sqrt{x_0^2 + \frac{v_0^2}{\omega_n^2}} \qquad \phi = \arctan\left(\frac{\omega_n x_0}{v_0}\right)$$

Particular solution for excitation $F(t) = F_0 \cos\omega t$

$$x_p(t) = \frac{F_0 / m}{\sqrt{\left(\omega_n^2 - \omega^2\right)^2 + \left(2\zeta\omega_n\omega\right)^2}}\cos\left(\omega t - \arctan\frac{2\zeta\omega_n\omega}{\omega_n^2 - \omega^2}\right)$$

For base excitation $y(t) = Y\sin\omega_b t$

$$x_p(t) = \omega_n Y\sqrt{\frac{\omega_n^2 + \left(2\zeta\omega_b\right)^2}{\left(\omega_n^2 - \omega_b^2\right)^2 + \left(2\zeta\omega_n\omega_b\right)^2}}\cos\left(\omega_b t - \arctan\frac{2\zeta\omega_n\omega_b}{\omega_n^2 - \omega_b^2} - \arctan\frac{\omega_n}{2\zeta\omega_b}\right)$$

$$X = Y\sqrt{\frac{1+\left(2\zeta r\right)^2}{\left(1-r^2\right)^2 + \left(2\zeta r\right)^2}} \qquad \frac{F_T}{kY} = r^2\sqrt{\frac{1+\left(2\zeta r\right)^2}{\left(1-r^2\right)^2 + \left(2\zeta r\right)^2}} \quad , \quad r = \frac{\omega_b}{\omega_n}$$

Rotating unbalance

$$m\ddot{x} + c\dot{x} + kx = m_0 e\omega_r^2 \sin\omega_r t$$

Momentum

$$\mathbf{p} = m\mathbf{v}$$

Conservation of energy

$T_1 + V_1 = T_2 + V_2$

T = *kinetic energy*

V = *potential energy*

Generally:

$$T = \frac{1}{2}m\dot{x}^2$$

$$T = \frac{1}{2}I\dot{\theta}^2$$

$$U = \frac{1}{2}kx^2$$

$$W_{1\to 2} = \int_{x_1}^{x_2} c\dot{y}\,dy$$

$$\frac{d\left(T+U\right)}{dt} = 0$$