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T-79.4101 Discrete Models and Search (5 cr)
Exam April 8, 2015

Write down on each answer sheet:
- Your name, degree program, and student number
- The text: "T-79.4101 Discrete Models and Search 8.4.2015
- The total number of answer sheets you are submitting for grading

Note: You can write down your answers in either Finnish, Swedish, or English. Calculators are allowed.

1. (a) Consider the constraint satisfaction problem (CSP)

$$\langle C_1(x,y,z), C_2(x,z); x \in \{1,2,3\}, y \in \{1,2,3\}, z \in \{1,2,3\}\rangle,$$

where $C_1 = \{(2,2,1),(2,1,2),(1,3,3)\}$ and $C_2 = \{(1,1),(2,2),(3,3)\}$. Apply the projection rule repeatedly until the CSP becomes hyper-arc consistent. (Note: for each application of the projection rule give the constraint and variable that was used and its domain after the application of the rule.) *(5 points)*

(b) Are the following two CSPs equivalent wrt. $\{x,y\}$:

$$\langle C_1(x,y,z); x \in D, y \in D, z \in D\rangle, \text{ and } \langle C_2(x,z); x \in D, z \in D\rangle,$$

where $D = \{1,2,3\}$, $C_1 = \{(1,2,3),(1,3,2),(2,1,3),(2,3,1),(3,1,2),(3,2,1)\}$ and $C_2 = \{(1,2),(1,3),(2,1),(2,3),(3,1),(3,3)\}$?
Justify your answer either by giving a counter-example, or a proof for the equivalence. *(5 points)*

2. (a) Express the condition "if $x+2>0$ then $y-4 \geq 0$" as a set of linear constraints, where x,y are integers such that $-8 \leq x,y \leq 8$. Explain your encoding. *(5 points)*

(b) Consider the following integer program: max $-2x_1 + 5x_2$ s.t.

$$\begin{aligned} &-x_1 + 3x_2 \geq 3 \\ &x_1 + x_2 \leq 7 \\ &x_2 \geq 0 \\ &x_1, x_2 \text{ are integers} \end{aligned}$$

Give the linear relaxation of the problem, transform the linear relaxation to the Simplex tableau form and give a basic feasible solution for the relaxation in the Simplex tableau form. Is the solution you gave optimal? Justify your answer using the Simplex tableau. *(5 points)*

3. (a) Simulate DPLL (draw a DPLL search tree) on the input CNF formula consisting of the clauses

$$(x_1 \vee x_2), (\neg x_1 \vee x_2), (\neg x_1 \vee \neg x_2), (x_1 \vee \neg x_3 \vee \neg x_4), (x_1 \vee x_3 \vee \neg x_4), (\neg x_2 \vee \neg x_3 \vee x_4), (\neg x_2 \vee x_3 \vee x_4).$$

Use any branching heuristic you want. Clearly denote which variable assignments are made by branching and which by unit propagation in the search tree. Is the CNF formula satisfiable? *(5 points)*

(b) Consider the following set of clauses:

$c1:$	$x_1 \vee x_{31} \vee \neg x_2$	$c2:$	$x_1 \vee \neg x_3$
$c3:$	$x_2 \vee x_3 \vee x_4$	$c4:$	$\neg x_4 \vee \neg x_5$
$c5:$	$x_{21} \vee \neg x_4 \vee \neg x_6$	$c6:$	$x_5 \vee x_6$

Assume decision assignments $x_{21} = 0@2$ and $x_{31} = 0@3$. Moreover, assume the current decision assignment to be $x_1 = 0@5$. (i) Draw the resulting implication graph. Does it yield a conflict? (ii) Which node in the implication graph is the 1-UIP (first unique implication point)? (iii) Give the conflict clause corresponding to the 1-UIP cut. *(5 points)*

4. Consider the following NP-complete NUMBER SET BIPARTITION problem: *(10 points)*

INSTANCE: A set of $2n$ natural numbers $A = \{a_1, \ldots, a_{2n}\} \subseteq \mathbb{N}$.
QUESTION: Is there a subset $B = \{b_1, \ldots, b_n\} \subseteq A$ containing exactly half the numbers and such that

$$\sum_{i=1}^{n} b_i = \frac{1}{2}\sum_{j=1}^{2n} a_j \quad ?$$

Formulate this task as an optimization problem by designing an appropriate objective function, and present pseudocode for the search for good bipartitions according to your criteria using *simulated annealing*.

Explain the following in detail: (a) what are the candidate solutions considered by your method and what is their neighborhood relation, (b) how does one choose the next solution for consideration from the neighborhood of a given candidate solution, and (c) how does one choose the initial candidate solution for the computation.