## Examination

School of Electrical Engineering
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## Answers briefly:

1. a) X is memoryless and thus remaining mean holding time is same as $E[X]=2 \mathrm{~min}$.
b) Again both are memoryless and time until either of the calls ends is the minimum of two i.i.d. exponential random variables, and the mean time until that is $E\left[\min \left(X_{1}, X_{2}\right)\right]=1 /(2 \lambda)=1$ min.

Grading: Each question gave 3 points. If memoryless property is not mentioned -1 p , and -2 p in part-b if minimum property is not mentioned.
2. This is about applying Little's result. By Little, arrival rate of customers at server is $\lambda=$ $N_{s} / T_{s}=21 / \mathrm{min}$. On the other hand, total delay $T_{t} o t=N_{t} o t / \lambda=10 / 2=51 / \mathrm{min}$ since system is stable.

Grading: Maximum 6p. Partial results were also given for incorrect answers. Using Little's result in some way, although incorrect, could give you max 3p. On the other hand, just writing the correct answer and no explanation, then max is 3 p.
3. This is the $\mathrm{M} / \mathrm{M} / 1$-FIFO queue.
a) $\rho=\lambda / \mu=8$ This is of course too high and was due to an error in link capacity, it should have been $C=100 \mathrm{Mbps}$.
b) using LBE's, $\pi_{n}=\rho^{n}(1-\rho)$ see M/M/1 lecture, must derive the stability condition $\rho<1$ ! c) Mean waiting time $=$ mean total delay $(1 /(\mu-\lambda)$, from Little) - service time ( 1 ms ). However, with the given load, $\rho=8$, since the system is unstable $\rho>1$, the total delay and hence the mean waiting time are infinite.

Grading: $\mathrm{a}=\max 1$ point, $\mathrm{b}=\max 3$ points, $\mathrm{c}=\max 2$ points. However, for max points it is not enough in part-b to just argue that the system is unstable, the stability condition must be derived, i.e., the equilibrium distribution needs to be solved. Partial points were given quite generously even with incorrectly calculated loads as long as the calculations seemed reasonable.
4. a) 2 servers and 1 waiting place. Thus, BD-process with states $0, \ldots, 3$, transition rate up $=\lambda$ and rate down in each state $\mu_{1}=\mu, \mu_{2}=2 \mu$ and $\mu_{3}=2 \mu$.
b) use LBE's to derive $\pi_{i}, i=1, \ldots, 3$
c) by PASTA, probability that arriving customer does not wait is $\pi_{0}+\pi_{1}=8 / 11$

Grading: a) $1 \mathrm{p}, \mathrm{b}$ ) $3 \mathrm{p}, \mathrm{c}$ ) 2 p (must justify by PASTA property!, if not -1 p ). Partial points were given for partially correct solutions. Even if your BD-process was wrong but you solved the steady state distribution correctly for the wrong model, I gave some points, max 3p.
5. See Demo 9, problem 4. a) $\phi(\mathbf{x})=1-\left(1-x_{1} x_{2}\right)\left(1-x_{1} x_{3}\right)\left(1-x_{2} x_{3}\right)$
b) $A=3 \cdot 0.9^{2}-2 \cdot 0.9^{3}=0.972$

Grading: a) 3 p (max 1 p , if no explanation, if explanation correct but structure wrong max 2p) b) 3 p (correct availability of one link $0.9,1 \mathrm{p}$, and calculation of mean availability from simplified structure function $\max 2 p$ ).

