

Answers briefly:

1. This is about applying Little's law.
 - a) 6 customers
 - b) 5.2 customers (total number minus number in service)
 - c) 0.8 customers (same as load of the system)
 - d) 20 customers (system is stable so flow in = flow out)

Grading: Each question gave 1.5 points and for the final points, the total was rounded to closest integer.

2. Here you need the memoryless property of exp-distribution and distribution of the minimum of exp-random variables.
 $E[Z_1] = 4$ (3+minimum of two exp-distributed rv's, memoryless after 3!)
 $E[Z_2] = 6$ (memoryless after departure of first, $Z_2 = Z_1 + \exp(1/2)$)

Grading: For full points, a complete answer with explicit logical reasoning was expected. If you only gave the correct answer and no justification, then maximum was 2 points. If memoryless property was not EXPLICITLY mentioned, then I took -1 point even if the answer was correct. Partial points were given for partly correct answers.

3. This is the M/M/1-PS queue.
 - a) $\rho = \lambda/\mu = 0.9$
 - b) using LBE's, $\pi_n = \rho^n(1 - \rho)$
 - c) 1Mbit/s (thput = mean SIZE/mean delay)

Grading: a = max 1 point, b = max 3 points, c = max 2 points.

4. a) 3 servers and 1 waiting place. Thus, BD-process with states $0, \dots, 4$, transition rate up = λ and rate down in each state $\mu_1 = \mu$, $\mu_2 = 2\mu$, $\mu_3 = 3\mu$ and $\mu_4 = 3\mu$.
 - b) use LBE's to derive $\pi_i, i = 1, \dots, 4$
 - c) by PASTA probability that arriving customer waits is $\pi_3 = 3/49$

Grading: a) 1p, b) 3p, c) 2p (must mention PASTA property!). Partial points were given for partially correct solutions. Even if your BD-process was wrong but you solved the steady state distribution correctly for the wrong model, I gave some points, max 2p.

5. a) $\phi(\mathbf{x}) = 1 - (1 - x_3(1 - (1 - x_1)(1 - x_2)))(1 - x_4)$
 - b) $A_1 = A_2 = 2/3, A_3 = 1, A_4 = 1/2$ and availability of the system $A_s = 1 - (1 - A_1)(1 - A_2)(1 - A_4) = 17/18$

Grading: a) 3p (partial points given if your solution was even close), b) availability of the components 2p and availability of the system 1p (minor miscalculations in obtaining the system availability were ignored).