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Department of Computer Science
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ICS-C2000 Theoretical Computer Science — Spring 2015

Exam April 8th, 2015, at 16:30–19:30

Use of calculators is not allowed in the exam.
Note: if you have not completed your computerized home assignments, your exam will not be graded.

1. (a) Design a deterministic finite state machine (i.e., finite automaton) that recognizes the language
$$\{w \in \{a,b\}^* \mid w \text{ starts with the substring } bab\}.$$
   (b) Design a deterministic finite state machine that recognizes the language
$$\{w \in \{a,b\}^* \mid w \text{ ends with the substring } bab\}.$$
   (c) Design a deterministic finite state machine that recognizes the language
$$\{w \in \{a,b\}^* \mid w \text{ does not contain the substring } bab\}.$$
   (d) Design a non-deterministic finite state machine that recognizes the language
$$\{w \in \{0,1\}^* \mid w \text{ contains the substring } 111 \text{ or } 011 \text{ (or both)}\}.$$
   Give a deterministic version of your machine recognizing the same language.

   10 points

2. (a) Give a regular expression that describes the language
$$L = \{w \in \{a,b\}^* \mid w \text{ begins and ends with different symbols}\}$$
   (b) Consider the regular expression $(0 \cup 1)^*1(0 \cup 1)$ over the alphabet $\{0,1\}$. Give the deterministic finite state machine with *minimal number of states* that recognizes the language described by the regular expression.
   (c) Give a regular expression that desribes the language
$$L = \{w \in \{a,b\}^* \mid w \text{ does not contain the substring } bab\}$$
   Hint: it may be a good idea to build the expression based on the deterministic finite state machine in the previous assignment.

   10 points

3. Consider the language
$$L = \{a^i(ca)^j b^k \mid i,j,k \geq 0 \text{ and } k = i + j\}$$
over the alphabet $\{a,b,c\}$.

(a) Prove that the language is not regular.
(b) Design a contex-free grammar that describes the language.
(c) Give a parse tree for the string $acacabbb$ in your grammar.
(d) Is your grammar in Chomsky normal form? Justify your answer.
(e) Is your grammar an LL(1) grammar? Justify your answer with few sentences.

12 points

4. Design a deterministic one-tape Turing machine that recognizes the language

$$L = \{wcw \mid w \in \{a, b\}^*\}$$

over the alphabet $\{a, b, c\}$.

Describe, with at most 10 sentences, how your machine works (that is, its design idea). Also show its computations on the inputs bcb and $acab$.

Does your machine also decide the language? (In terms of "Orposen pruju", is your machine total?)

8 points

5. (a) Desribe in your own words (with at most 5 sentences), what "Church–Turing thesis" is.
(b) Define the concepts "Turing-recognizable language" and "Turing-decidable language" ("recursively enumerable language" and "recursive language" in "Orposen pruju").
(c) Consider the language

$$\{x \in \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}^* \mid x \text{ is a product of two prime numbers}\}.$$

For instance, the string 15 is in the language as $3 \times 5 = 15$ but 16 is not in the language. Is the language Turing-decidable? Justify your answer.
(d) Prove the following claim either correct or incorrect:

Let L_1 and L_2 be languages over an alphabet Σ. If L_1 is context-free and L_2 is Turing-decidable, then also the language $L_1 \cap L_2$ is context-free.

10 points

6. Consider the following decision problem:

Given a Turing machine M. Is the language recognized by the machine a context-free language?

It can be described as a language

$$L_{\text{cfg}} = \{\langle M \rangle \mid M \text{ is a Turing machine and } L(M) \text{ is a context-free language}\}.$$

Prove that the language L_{cfg} is undecidable. If you use Rice's theorem (you don't have to), also give a definition for Rice's theorem as well as for the related concepts of "semantic property" and "trivial semantic property".

9 points

7. At what time did you finish answering the exam questions?

1 points