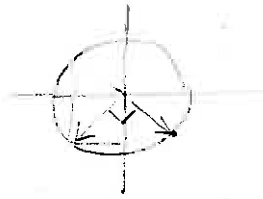
VK 1   7/8/2015



1)

$$2ie^{-i\frac{5\pi}{4}} = 2i\Big(\underbrace{\cos\tfrac{5\pi}{4}}_{-1/\sqrt{2}} - i\underbrace{\sin\tfrac{5\pi}{4}}_{+1/\sqrt{2}}\Big)$$

$$= 2i\frac{1}{\sqrt{2}}(-1-i) = \underline{\sqrt{2}(1-i)}$$

$$(5+5i)^3 = 5^3(1+i)^3 = 125\left(\sqrt{2}e^{i\frac{\pi}{4}}\right)^3 = 125\cdot 2\sqrt{2}\left(e^{i\frac{3\pi}{4}}\right)$$

$$= \underbrace{250\sqrt{2}}_{=r}\Big(\cos\underbrace{\left(\tfrac{3\pi}{4}\right)}_{=\theta} + i\sin\left(\tfrac{5\pi}{4}\right)\Big)$$

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4) a) Bruge testen

$$\left|\frac{z_{n+1}}{z_n}\right| = \left|\frac{\frac{z^{n+1}}{(n+1)^2}}{\frac{z^n}{n^2}}\right| = \left(\frac{n}{n+1}\right)^2|z| \longrightarrow |z|$$

for $n \to \infty$

Rækkens konvergensområde er $\underline{|z| < 1}$

b) Husk definitionen $\binom{n}{2} = \frac{n!}{2!(n-2)!}$

Skriv rækken

$$\sum_{n=2}^{\infty}\binom{n}{2}z^n = \binom{2}{2}z^2 + \binom{3}{2}z^3 + \binom{4}{2}z^4 + \binom{5}{2}z^5 + \dots$$

$$= z^2 + 3z^3 + 6z^4 + 10z^5 + \dots$$

$$= \frac{z^2}{2}\Big(\underbrace{2 + 6z^3 + 12z^4 + 20z^3 + \dots}_{\frac{d^2}{dz^2}\sum_{n=0}^{\infty} z^n}\Big)$$

Geom. række konvergerer for $|z| < 1$, dvs derivation må tillades.

3)

$$\ln 1 = \ln|1| + i(\operatorname{Arg}(1) + n2\pi)$$
$$= 0 + i(0 + n2\pi) = \underline{ni2\pi \quad , n \in \mathbb{Z}}$$

$$e^{\ln 1} = 1$$

$$\ln(-e^{-i}) = \ln|-e^{-i}| + i(\operatorname{Arg}(-e^{-i}) + n2\pi)$$
$$= \underbrace{\ln|1|}_{=0} + i(\operatorname{Arg}(e^{i\pi}e^{-i}) + n2\pi)$$
$$= 0 + i(\operatorname{Arg}(e^{i(\pi-1)}) + n2\pi)$$
$$= \underline{i(\pi - 1 + n2\pi) \quad , n \in \mathbb{Z}}$$

2)

$$u = x^2 - y^2 - x \qquad (*)$$
$$u_x = 2x - 1 \underset{\text{C-Riemann}}{=} v_y \Rightarrow v = \int (2x-1)\,dy = 2xy - y + k(x)$$

Toisesta CR-yhtälöstä $(u_y = -v_x)$ saadaan

$$u_y = -(2y - k'(x)) = -2y + k'(x)$$

Toisaalta $(*)$ antaa:

$$u_y = -2y$$

josta $k'(x) = 0$ eli $k = \text{vakio}$

Siten $\underline{f(z) = x^2 - y^2 - x + i(2xy - y + k)}$