

Suorita 5 tehtävää tehtävistä 1-6! Laskimen muistiin ei saa olla talletettu kirjoitettua informaatiota. Tehtäväpaperin lopussa on apuvälineitä tenttiä varten.

- 1) a) Which are the allowed quantum mechanical values of the length of the angular momentum vector? (2p) b) A particle having the total angular momentum quantum number J is placed in homogeneous external magnetic field. How does the directional quantization angular momentum influence the splitting of energy levels? (2p) c) If a particle with spin $S = 3/2$ couples to a particle with spin $S = 1/2$, what are the possible values for the spin of the resulting particle? (2p)
- 2) The spectrum of figure 2.1 is caused by a molecule that includes ^{13}C nuclei. These nuclei are in interaction with a number of ^1H isotopes (protons). a) What is the interaction mechanism? Give reason for this mechanism! (3p) b) Explain how many protons are there interacting with ^{13}C and why the spectrum has intensity ratio of 1:3:3:1! (3p)

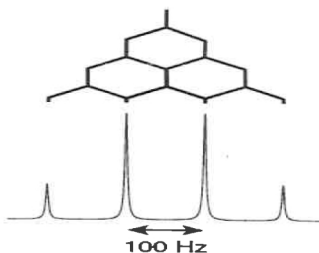


Figure 2.1

- 3) A system of spins is in thermal equilibrium at temperature T . a) The system is placed in a homogeneous static external magnetic field. Give the equation of induced magnetization as a function of time counted from the moment the field is switched on. Draw a picture of the magnetization as a function of time! (Give the equation in terms of equilibrium magnetization M_{eq}^{nuc} and appropriate relaxation time.) (3p) b) The magnetic field is now turned off. Give the magnetization as a function of time and draw a picture like in point a). (3p)

4. An NMR signal is performed on isolated proton spins.

a) Suppose that the peak r.f. field in the centre of the sample is $B_{RF} = 4.697$ mT, and that a pulse of duration $\tau_p = 5 \mu\text{s}$ and phase $\theta_p = 0$ is applied. What is the flip angle of the pulse in the centre of the sample? If the magnetization vector before the pulse is $M = e_z$, what is the magnetization vector after the pulse? Ignore the off-resonance effects. (2p)

b) At the edge of the sample, the peak r.f. field is only $B_{RF} = 4.228$ mT. If the magnetization vector at the edge of the sample before the pulse is $M = e_z$, what is the magnetization vector at the edge of the sample after the pulse? What is the angle between the magnetization vectors at the edge of the sample and at the centre of the sample, after the pulse? (2p)

c) Now suppose, that the single r.f. pulse is replaced by a sequence of three pulses of durations $2.5 \mu\text{s}$, $5 \mu\text{s}$ and $2.5 \mu\text{s}$, with phases $0, \pi/2$ and 0 , respectively. This three sequence pulse is an example of a composite pulse. If the magnetization vector before the pulse is $M = e_z$, what is the magnetization vector at the centre of the sample after the pulse? (2p)

5. On the basis of the given state populations related to a closed cycle of spin states (Figure 5.1) reconstruct the corresponding values of a) the magnetization vectors (3p) and b) density matrix (3p) for each intermediate state.

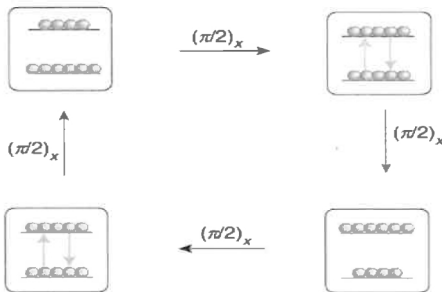


Figure 5.1

6. Consider a x-pulse which transfers spin population in an inversion state by rotating it around suitable axis. a) What is the density matrix of the system before the pulse? (2p) b) How does the density matrix change during this pulse? (4p) (Hint: Assume that the spins are initially in equilibrium with the external magnetic field in temperature T . In thermal equilibrium the population ratio obeys Boltzmann equation.)