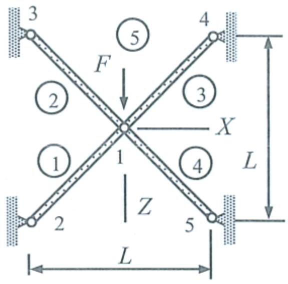
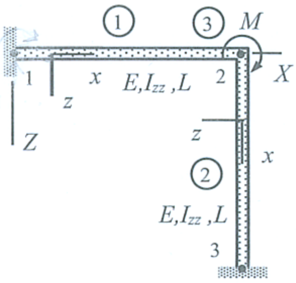
Kul-49.3300 Finite element method I, exam, 31.08.2015

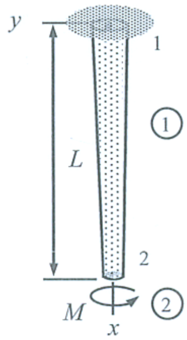
1. Determine horizontal and vertical displacements of node 1 of the bar structure of the figure. The cross-sectional area of all the bars is $\sqrt{2}A$ and Young's modulus of the material is E. Use the variational form of element contributions (four elements) and principle of virtual work.



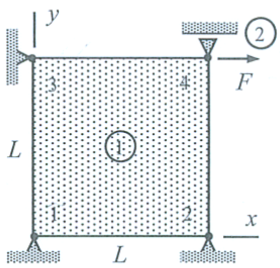
2. Determine the rotation $\theta_{Y2} = \mathrm{a}$ at node 2 of the structure loaded by a point moment (magnitude M) acting on node 2. Use two Bernoulli beam elements (1,2) of equal length and a point moment element (3). Assume that the beams are rigid in the axial directions. Young's modulus of the material E and the second moment of area I_{zz} are constants.



3. Consider the torsion bar (1) of the figure loaded by torque M (2) acting on the free end. Determine the rotation $\theta_{x2} = \mathrm{a}$ at the free end if the diameter of the circular cross-section is given by $d = t(2 - x/L)^{1/4}$. Material property G is constant. Start with the virtual work density $\delta w^{\mathrm{int}} = -\delta\phi_{,x} G I_{rr} \phi_{,x}$ and use linear approximation to rotation (a linear two-node element). The definition of cross-section moment is $I_{rr} = \int r^2 dA$.



4. A square thin slab (1) is loaded by a point force (2) as shown in the figure. Derive the relationship between the force magnitude F and it's displacement $u_{x4} = \mathrm{a}$. Young's modulus E, Poisson ratio ν, and thickness of the slab t are constants. The external distributed forces are zeros. Assume plane-stress conditions, start with the virtual work density, and assume bilinear approximation.



5. A Kirchhoff plate, loaded by its own weight, is simply supported on two edges and free on the other two edges as shown in the figure. Use the one-parameter approximation $w(x) = \mathrm{a}(1 - x/L)(x/L)$ to determine the displacement at the center point. Problem parameters E, ν, ρ and t are constants.