Aalto University
School of Science

PHYS-E0413 Theoretical Mechanics, Autumn 2015, Midterm exam 2.

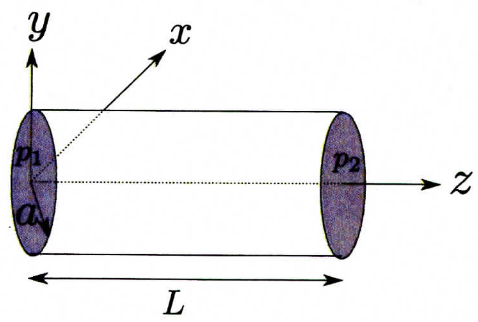
1. Explain the key concepts of mathematically describing fluid flow (not the full derivation of the Navier-Stokes equation).

2. Bernoulli's theorem:

   (a) Starting from the Euler's equation,
   $$\frac{\partial}{\partial t}\vec{v} + (\vec{v} \cdot \nabla)\vec{v} = -\frac{1}{\rho}\nabla p + \vec{f},$$
   using the vector identity
   $$(\vec{v} \cdot \nabla)\vec{v} \equiv \nabla(\frac{1}{2}v^2) - [\vec{v} \times (\nabla \times \vec{v})],$$
   derive Bernoulli's theorem.

   (b) Give an example of its application.

3. A fluid flows in a long pipe with a circular cross section of radius a. Assume that the flow is laminar, stationary, and incompressible and that there are no volumetric forces present. The pressure at $z = 0$ is p_1 and at $z = L$ the pressure is p_2. The density of the fluid is ρ and the kinematic viscosity ν.



   (a) Solve for the pressure field p and the velocity field v of the fluid.

   (b) Calculate the drag force the fluid exerts on the wall of the cylinder for $0 \leq z \leq L$.

   (c) Calculate the mass flow rate through the pipe.

4. Give an example of a strain tensor for which there is

   (a) an increase in volume.

   (b) extension in the z-direction, but overall decrease in volume.

   (c) shear strain, but no volume change.

   For the strain tensor of (a) write down the corresponding stress tensor for an isotropic solid given the Lamé coefficients μ and λ.