Aalto University School of Science
Systems Analysis Laboratory

MS-E2139 Nonlinear Programming Kimmo Berg *Exam, 10.12.2015*

Note: problems 4 and 5 are **optional** for those who started the programming project work.

1. Explain shortly the following notions:
   a) supporting hyperplane
   b) lower-level set
   c) condition number
   d) weak duality
   e) Maratos effect
   f) complementary slackness condition

2. a) Define local and global optimality. Explain what the necessary conditions and sufficient conditions mean.
   b) Define a convex set and a convex function.
   c) Define the convex optimization problem. Prove that the local minimum of a convex unconstrained problem is also global.
   d) Write out the necessary and sufficient conditions (not KKT!) to the convex optimization problem. Define the needed notions.
   e) Solve the following problem using the above conditions

$$\begin{array}{ll} \min & \|x-(1,1)\|_1 = |x_1-1|+|x_2-1| \\ \text{s.t.} & x_1 \leq 0. \end{array}$$

   Prove that the problem has a solution.

3. a) Solve the problem

$$\begin{array}{ll} \min & x_1 x_2 \\ \text{s.t.} & x_1 \geq 1, \quad x_2 \geq 1 \\ & 2x_1 + x_2 \geq 6. \end{array}$$

   Find all local minimum points. Find a KKT point that is not a local minimum.
   b) Solve the problem

$$\begin{array}{ll} \min & x_1 \\ \text{s.t.} & x_2 \leq 0, \quad x_1^2 - x_2 \leq 0. \end{array}$$

PLEASE TURN!

4.* a) Compare the central methods of unconstrained multidimensional optimization.

b) Derive the update equations in the primal-dual interior point algorithm for the general form LP problem using logarithmic barrier function.

5.* a) Explain how the reduced gradient method works.

b) Explain how the active set method works and solve using it the problem:

$$\begin{array}{ll} \min & \frac{1}{2}x^T Hx + c^T x \\ \text{s.t.} & Ax \geq b, \end{array}$$

where

$$H = \begin{pmatrix} 2 & -1 \\ -1 & 2 \end{pmatrix}, \; c = \begin{pmatrix} -4 \\ -1 \end{pmatrix}, \; A = \begin{pmatrix} 1 & 0 \\ 0 & 1 \\ -1 & 0 \\ 0 & -1 \\ -4 & -6 \end{pmatrix}, \; b = \begin{pmatrix} 0 \\ 0 \\ -5 \\ -5 \\ -35 \end{pmatrix}.$$

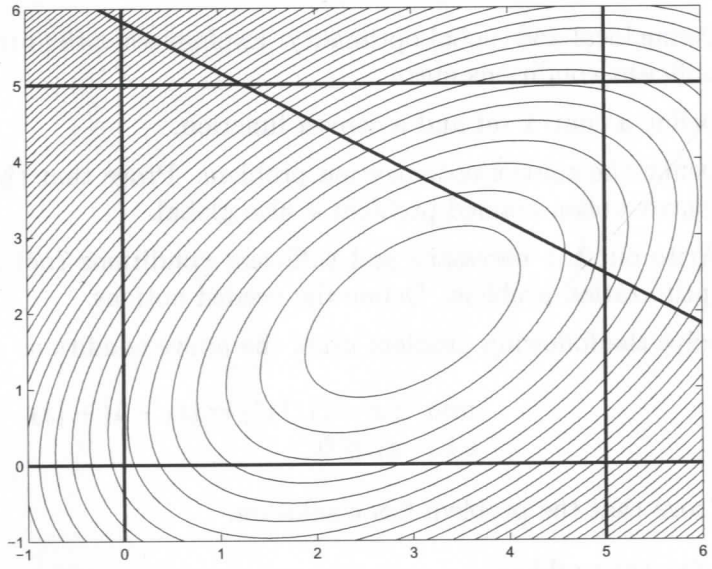

Figure 1: Problem 5b.

Start at point $x = (5\ 0)^T$ and active constraint $x_1 \leq 5$. Explain all the steps. You can use the figure in the steps.