Problems

1. The sphere model in MEG

Outside a spherically symmetric conductor with isotropic electrical conductivity, there is no contribution from the volume currents to the radial component of the magnetic field (B_r). Therefore, B_r for a current dipole can be computed directly from the Ampère-Laplace law:

$$\vec{B} = \frac{\mu_0}{4\pi} \frac{\vec{Q} \times \vec{R}}{R^3},$$

where $\vec{Q}$ is the current dipole moment, $\vec{R} = \vec{r} - \vec{r}_Q$ is the vector from the location of the dipole ($\vec{r}_Q$) to the field point ($\vec{r}$), and $\mu_0 = 4\pi \cdot 10^{-7}$ H/m is the magnetic permeability of vacuum. Assume that:

(i) The spherically symmetric conductor model is a good approximation to the conductivity geometry of the head and that the radius of curvature of the scalp surface is 90 mm.

(ii) The equivalent source for brain activity is a tangential current dipole with amplitude Q located at depth $d > 0$ from the scalp surface.

(iii) MEG is measured with point like sensors outside the head detecting the radial field component at distance D from the scalp.

What are the maximum amplitudes of the magnetic field detected when the source and sensor parameters are:

	(a)	(b)	(c)	(d)
Q / nAm	30	30	30	30
d / mm	20	20	40	40
D / mm	20	5	20	5

2. Magnetic Resonance Imaging

(a) Describe the process of slice selection and two-dimensional Fourier encoding of MRI data. Use a schematic of the pulse sequence to illustrate your answer.

(b) A tissue has T1 = 1000 ms, T2 = 100 ms, and T2* = 50 ms. A spin echo (180° second RF pulse) experiment starts from thermal equilibrium, flip angle = 90°, and TE = 100 ms. What is the signal intensity of the spin echo in relation to the equilibrium magnetization?

3. X-Ray Computed Tomography (CT)

(a) Describe the physical principles behind X-ray computed tomography (CT), the interaction between x-rays and tissue, and X-ray generation and detection.

(b) Formalize the image reconstruction problem as a minimization problem and discuss sources of error in CT reconstructions.

4. Functional Imaging of the Brain

Discuss the characteristics and use of MEG, EEG, and fMRI in studying human brain function. Consider the following aspects:

(i) What is the physiological origin of the signal detected and what is the timescale of changes in this signal?

(ii) How are the signals measured / images acquired?

(iii) What are the sources of noise?

(iv) What is the typical spatial resolution of each imaging modality?

(v) Is the spatial (image) reconstruction problem (inverse problem) unique? Why?

(vi) Which combinations of MEG, EEG, and fMRI data can be acquired simultaneously? What kind of additional technical challenges do such combined acquisitions pose?

(vii) How will each imaging modality benefit from the availability of individual anatomical MRI data?

Each problem yields a maximum of 6/30 points.
Returned assignments yield maximally another 6 points, totaling 4 x 6 + 6 = 30 points.