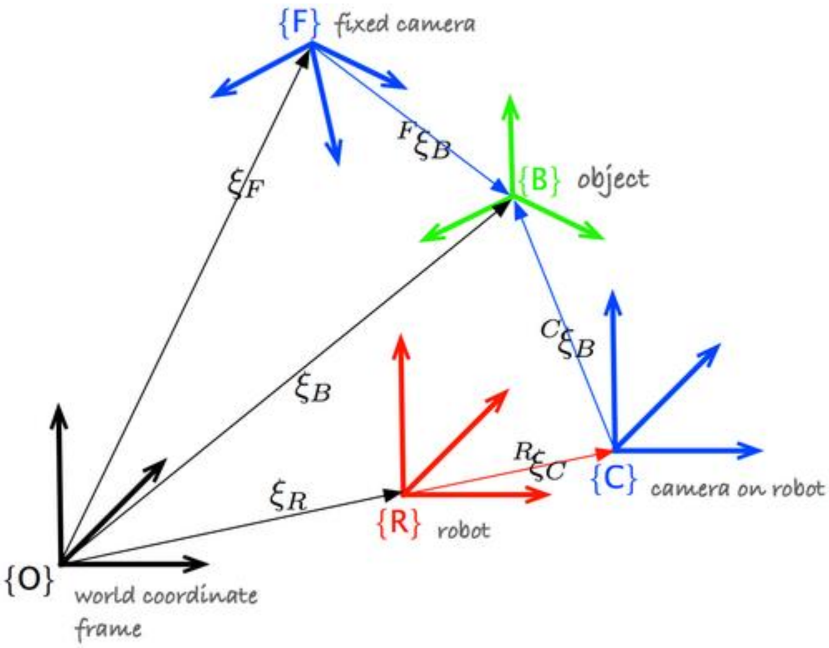
ELEC-C1320 – Robotiikka,   Exam 10.12.2015

It is allowed to use a calculator and a book of mathematical equations (e.g. MAOL) in the exam.

1. In the figure below, known relative transformations between some of the key coordinate frames in a robot cell are described with the ξ-symbol. Based on the figure, give all the kinematic paths to describe the position and orientation of the robot {R} with respect to the world coordinate frame {O}. (15 points)



Solution:

There are three paths that link coordinate frame {R} with the world coordinate frame {O}: *note that here "-" sign means that you take first the inverse of the relative pose transformation before compounding it with the neighboring transformation(s)*

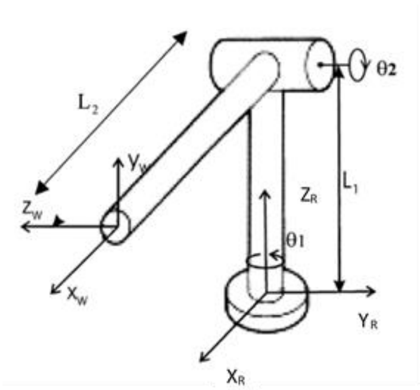
$^O\xi_R = {}^O\xi_R$

$^O\xi_R = \xi_B \ {-}^C\xi_B \ {-}^R\xi_C = \xi_B \ ^B\xi_C \ ^C\xi_R$

$^O\xi_R = \xi_F \ ^F\xi_B {-}^C\xi_B \ {-}^R\xi_C = \xi_F \ ^F\xi_B {}^B\xi_C \ ^C\xi_R$

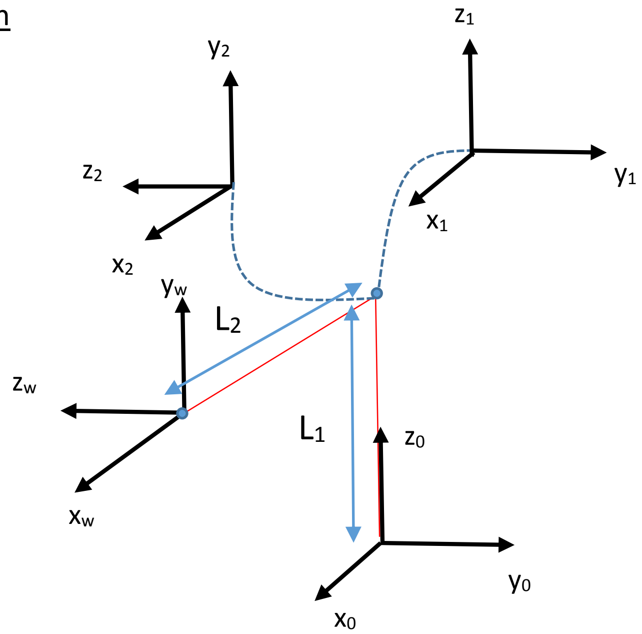
2. Make/create the forward kinematic model for the 2-degree of freedom manipulator shown in the figure below. In the figure, the manipulator is shown in its home/zero position (i.e. when the joint control angles Θ_1 and Θ_2 are zero, the upper arm is oriented horizontally above the x_R-axis). Both degrees-of-freedom (dof) are rotational (the first rotating the upper link on the horizontal plane, Θ_1, and the second tilting the upper link with respect to the horizontal plane, Θ_2). Positive directions of rotations are shown in the figure.

The forward kinematic model should describe the Wrist frame, W, position and orientation w.r.t the manipulator base frame, R, ${}^{R}\mathbf{T}_{W}$. It is your choice to use either the Standard or Modified DH-parameter convention. As an answer to the problem, give a drawing showing the link frames for each of the two links. Also, give the DH-parameters in a table. Note: remember also to give (if necessary) the additional base transformation and tool transformation (parameters/matrix) for describing the whole kinematic path from the robot base frame, R, up to the wrist/hand frame W. (18 points)



Solution:

We select here the Modified DH-parameter convention



Link	α_{i-1}	a_{i-1}	d_i	Θ_i	σ_i
1	0	0	0	Θ_1	0
2	90	0	0	Θ_2	0

In addition we will need both the base transformation to contribute for the height of the shoulder joint above the 0-frame, L_1 (along the z_0 axis)

$$\text{base transformation} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & L_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

and the tool transformation to move from the joint axis of the shoulder joint to the origin of the w-frame, L_2 (along the x_2 axis):

$$\text{tool/hand transformation } H = \begin{bmatrix} 1 & 0 & 0 & L_2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

3. For the same manipulator, which was introduced in problem 2, create the manipulator Jacobian matrix in symbolic form. The Jacobian matrix, $\boldsymbol{J}$, should map the velocities of the manipulator joints into the Cartesian velocities of the origin of the W-frame, i.e.: (17 points)

$$\begin{bmatrix} v_x \\ v_y \\ v_z \end{bmatrix} = \boldsymbol{J} \begin{bmatrix} \dot{\boldsymbol{\theta}}_1 \\ \dot{\boldsymbol{\theta}}_2 \end{bmatrix}$$

Solution:

For determining the Jacobian matrix we first form the arm matrix for the 2-link manipulator by using the forward kinematics solution from problem 2 and the symbolic form of the link matrix for the modified DH-parameter convention:

$$\begin{bmatrix} c\theta_i & -s\theta_i & 0 & a_{i-1} \\ s\theta_i c\alpha_{i-1} & c\theta_i c\alpha_{i-1} & -s\alpha_{i-1} & -s\alpha_{i-1} d_i \\ s\theta_i s\alpha_{i-1} & c\theta_i s\alpha_{i-1} & c\alpha_{i-1} & c\alpha_{i-1} d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

The link matrices are:

$$^0A_1 = \begin{bmatrix} c\theta_1 & -s\theta_1 & 0 & 0 \\ s\theta_1 & c\theta_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$^1A_2 = \begin{bmatrix} c\theta_2 & -s\theta_2 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ s\theta_2 & c\theta_2 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

after calculating them together we get:

$$^0T_2 = {}^0A_1 {}^1A_2 = \begin{bmatrix} c\theta_1 c\theta_2 & -c\theta_1 s\theta_2 & s\theta_1 & 0 \\ s\theta_1 c\theta_2 & -s\theta_1 s\theta_2 & -c\theta_1 & 0 \\ s\theta_2 & c\theta_2 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

and then we calculate the result and the tool transformation matrix together:

$$^0T_W = {}^0T_2 H = \begin{bmatrix} c\theta_1 c\theta_2 & -c\theta_1 s\theta_2 & s\theta_1 & c\theta_1 c\theta_2 L_2 \\ s\theta_1 c\theta_2 & -s\theta_1 s\theta_2 & -c\theta_1 & s\theta_1 c\theta_2 L_2 \\ s\theta_2 & c\theta_2 & 0 & s\theta_2 L_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

and finally multiply the base transformation matrix with the result from above:

$$^RT_W = {}^RT_0 {}^0T_W = \begin{bmatrix} c\theta_1 c\theta_2 & -c\theta_1 s\theta_2 & s\theta_1 & c\theta_1 c\theta_2 L_2 \\ s\theta_1 c\theta_2 & -s\theta_1 s\theta_2 & -c\theta_1 & s\theta_1 c\theta_2 L_2 \\ s\theta_2 & c\theta_2 & 0 & s\theta_2 L_2 + L_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

which is the forward kinematics solution in the form of manipulator arm matrix. We are now only interested in the x-, y- and z-coordinates of the origin of the W-frame, i.e. the tree top elements of the fourth column of the arm matrix.

x=$c\theta_1 c\theta_2 L_2$

y= $s\theta_1 c\theta_2 L_2$

z= $s\theta_2 L_2 + L_1$

The Jacobian matrix to map the joint velocities to the Cartesian velocity of the origin of the W-frame:

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \end{bmatrix} = \boldsymbol{J}\dot{\boldsymbol{q}} = \begin{bmatrix} \frac{dx}{d\theta_1} & \frac{dx}{d\theta_2} \\ \frac{dy}{d\theta_1} & \frac{dy}{d\theta_2} \\ \frac{dz}{d\theta_1} & \frac{dz}{d\theta_2} \end{bmatrix} \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \end{bmatrix} = \begin{bmatrix} -s\theta_1 c\theta_2 L_2 & -c\theta_1 s\theta_2 L_2 \\ c\theta_1 c\theta_2 L_2 & -s\theta_1 s\theta_2 L_2 \\ 0 & c\theta_2 L_2 \end{bmatrix} \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \end{bmatrix}$$

where $\boldsymbol{J}$ is the Jacobian matrix, i.e. solution to problem 3.

4. A weight of 3kg is fixed to the tip of last link (at the location of the origin of the W-frame) of the two-link planar manipulator, introduced in problem 2 and shown in the figure on the 2nd page. The task is to calculate torques affecting joints 1 and 2 (due to gravity) in two different configurations of the manipulator arm. To solve the problem here you must utilize the Jacobian matrix formed in problem 3. The joint configurations to be considered are: (15 points)

a) $\theta_1 = 0.0°, \theta_2 = 0.0°$

b) $\theta_1 = 0.0°, \theta_2 = 90.0°$

The lengths of the beams of the arm are $L_1 = 0.6m$, $L_2 = 0.5m$.
The links itself are assumed to be weightless.
The gravitational acceleration vector is pointing in the direction of negative z_R-axis and its value is 9.81 m/s^2.

Solution:

The joint torques to support the load are calculated by multiplying the gravity force vector with the transpose of the manipulator Jacobian matrix:

$$\boldsymbol{T} = \boldsymbol{J}^T \boldsymbol{f}$$

So, first calculate the transpose of the Jacobian matrix, which was formed as the solution of problem 3:

$$\boldsymbol{J^T} = \begin{bmatrix} -s\theta_1 c\theta_2 L_2 & -c\theta_1 s\theta_2 L_2 \\ c\theta_1 c\theta_2 L_2 & -s\theta_1 s\theta_2 L_2 \\ 0 & c\theta_2 L_2 \end{bmatrix}^T = \begin{bmatrix} -s\theta_1 c\theta_2 L_2 & c\theta_1 c\theta_2 L_2 & 0 \\ -c\theta_1 s\theta_2 L_2 & -s\theta_1 s\theta_2 L_2 & c\theta_2 L_2 \end{bmatrix}$$

and the force/wrench vector due to the 3 kg weight (gravity) force pulling the tip of the arm in the direction of negative z_R-axis is: $\boldsymbol{f} = \begin{bmatrix} 0.0 \\ 0.0 \\ -9.81 * 3.0N \end{bmatrix}$

a) In the first case, the manipulator joint configuration was given as $\theta_1 = 0.0°, \theta_2 = 0.0°$

So, the transpose of the manipulator Jacobian becomes:

$$\boldsymbol{J^T} = \begin{bmatrix} -s\theta_1 c\theta_2 L_2 & c\theta_1 c\theta_2 L_2 & 0 \\ -c\theta_1 s\theta_2 L_2 & -s\theta_1 s\theta_2 L_2 & c\theta_2 L_2 \end{bmatrix} = \begin{bmatrix} 0.0 & L_2 & 0.0 \\ 0.0 & 0.0 & L_2 \end{bmatrix}$$

And now we can calculate the joint torques, Q_1 and Q_2, required to support the load:

$$\boldsymbol{T} = \boldsymbol{J}^T \boldsymbol{f} = \begin{bmatrix} Q_1 \\ Q_2 \end{bmatrix} = \begin{bmatrix} 0.0 & 0.5 & 0.0 \\ 0.0 & 0.0 & 0.5 \end{bmatrix} \begin{bmatrix} 0.0 \\ 0.0 \\ -29.43N \end{bmatrix} = \begin{bmatrix} 0.0 \\ -14.7Nm \end{bmatrix}$$

b) In the second case, the manipulator joint configuration was given as $\theta_1 = 0.0°$, $\theta_2 = 90.0°$

So, the transpose of the manipulator Jacobian becomes:

$$\boldsymbol{J}^T = \begin{bmatrix} -s\theta_1 c\theta_2 L_2 & c\theta_1 c\theta_2 L_2 & 0 \\ -c\theta_1 s\theta_2 L_2 & -s\theta_1 s\theta_2 L_2 & c\theta_2 L_2 \end{bmatrix} = \begin{bmatrix} 0.0 & 0.0 & 0.0 \\ -L_2 & 0.0 & 0.0 \end{bmatrix}$$

And the joint torques, Q_1 and Q_2, required to support the load are:

$$\boldsymbol{T} = \boldsymbol{J}^T \boldsymbol{f} = \begin{bmatrix} Q_1 \\ Q_2 \end{bmatrix} = \begin{bmatrix} 0.0 & 0.0 & 0.0 \\ -0.5 & 0.0 & 0.0 \end{bmatrix} \begin{bmatrix} 0.0 \\ 0.0 \\ -29.43N \end{bmatrix} = \begin{bmatrix} 0.0 \\ 0.0 \end{bmatrix}$$

The result makes sense because when the upper arm points straight upwards there will be no level arm to generate torque to the joints when the force vector is parallel to the upwards pointing upper arm.

ELEC-C1320 Robotiikka - Equations

Link parameters and the corresponding elementary transformations as well as the symbolic form of the link matrix according to the **standard** Denavit and Hartenberg parameter convention:

$$^{j-1}\boldsymbol{A}_j(\theta_j, d_j, a_j, \alpha_j) = \boldsymbol{T}_{Rz}(\theta_j)\boldsymbol{T}_z(d_j)\boldsymbol{T}_x(a_j)\boldsymbol{T}_{Rx}(\alpha_j)$$

$$^{j-1}\boldsymbol{A}_j = \begin{pmatrix} \cos\theta_j & -\sin\theta_j \cos\alpha_j & \sin\theta_j \sin\alpha_j & a_j \cos\theta_j \\ \sin\theta_j & \cos\theta_j \cos\alpha_j & -\cos\theta_j \sin\alpha_j & a_j \sin\theta_i \\ 0 & \sin\alpha_j & \cos\alpha_j & d_j \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Link parameters and the corresponding elementary transformations as well as the symbolic form of the link matrix according to the **modified** Denavit and Hartenberg parameter convention:

$$^{j-1}\boldsymbol{A}_j = \boldsymbol{R}_x(\alpha_{j-1})\boldsymbol{T}_x(a_{j-1})\boldsymbol{R}_z(\theta_j)\boldsymbol{T}_z(d_j)$$

$$^{j-1}\boldsymbol{A}_j = \begin{bmatrix} c\theta_j & -s\theta_j & 0 & a_{j-1} \\ s\theta_j c\alpha_{j-1} & c\theta_j c\alpha_{j-1} & -s\alpha_{j-1} & -s\alpha_{j-1} d_j \\ s\theta_j s\alpha_{j-1} & c\theta_j s\alpha_{j-1} & c\alpha_{j-1} & c\alpha_{j-1} d_j \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Elementary rotation transformations (i.e. rotations about principal axis by Θ):

$$R_x(\theta) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{pmatrix}$$

$$R_y(\theta) = \begin{pmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{pmatrix}$$

$$R_z(\theta) = \begin{pmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Inverse of a 4x4 transformation matrix:

$$\boldsymbol{T}^{-1} = \begin{pmatrix} \boldsymbol{R} & \boldsymbol{t} \\ \boldsymbol{0}_{1\times 3} & 1 \end{pmatrix}^{-1} = \begin{pmatrix} \boldsymbol{R}^T & -\boldsymbol{R}^T\boldsymbol{t} \\ \boldsymbol{0}_{1\times 3} & 1 \end{pmatrix} \tag{2.21}$$

Derivation of trigonometric functions:

Dsinx = cosx

Dcosx = -sinx

Definition of (manipulator) Jacobian matrix:

If $\boldsymbol{y} = F(\boldsymbol{x})$ and $\boldsymbol{x} \in \mathbb{R}^n$ and $\boldsymbol{y} \in \mathbb{R}^m$ then the Jacobian is the $m \times n$ matrix

$$J = \frac{\partial F}{\partial \boldsymbol{x}} = \begin{pmatrix} \frac{\partial y_1}{\partial x_1} & \cdots & \frac{\partial y_1}{\partial x_n} \\ \vdots & \ddots & \vdots \\ \frac{\partial y_m}{\partial x_1} & \cdots & \frac{\partial y_m}{\partial x_n} \end{pmatrix}$$