

Allowed material: writing implements and a scientific calculator. You are not allowed to use any other material. There are some formulas and constants tabulated in last page of the exam. Justify the formulas you use in your answers and the intermediate steps you take. Introduce the meaning of the symbols within these formulas. In every problem, both the presentation and the contents are evaluated when grading the exam. Solve each problem on separate page.

*If there are any inconsistencies with this exam, the Finnish version is the authoritative version. It is important that you at least try each problem. Good luck!*

1. (a) Define the following terms/concepts using at most about 30 words / term. Using only formulas is not a sufficient answer. Considerably too long an answer will decrease the points awarded.

A. Central force    B. Kinetic energy    C. Mass

Answer the following question using at most about 200 words. Significantly overlong or poorly structured answer will not be awarded full points. You may use drawing to support your answer, but answering using only figures will not yield points.

- (b) We want maximize the moment of inertia of a flywheel, while its total mass needs to be kept as small as possible. How should the mass be distributed on the flywheel?
2. The mass of an electric car is 1000 kg. The maximum velocity of the car on a horizontal road is 125 km/h, whereas the maximum velocity drops to 115 km/h as the car goes up a hill whose slope is 5%. You may assume all frictional forces constant. How far up the hill can the car go, if has 54 kWh of energy to use and it is traveling at its maximum speed?
- (NB: 1% slope has an angle  $\alpha$  with respect to horizon such that  $\tan \alpha = 0.01$ )
3. A restoring force  $-kx$  and a conservative constant force  $F$  simultaneously affect a spring. Both forces are oriented along the  $x$ -axis.

- (a) Show that the net effect of the forces can be described with a potential function

$$U(x) = \frac{1}{2}kx^2 - Fx - \frac{F^2}{2k}.$$

- (b) Can the potential function describing the net effect of the forces have other functional forms as well, that is, is it unique? Justify.