Aalto University School of Electrical Engineering
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EXAM
17.2.2016

ELEC-E7240 Coding Methods

All answers must be motivated. In particular, a one-word answer for yes/no questions is not sufficient.

1. (6p.) Error-detecting codes.

   In the University of Wonderland student ID numbers are 6-digit numbers $a_0a_1a_2a_3a_4a_5$ (indeed, these are the *digits* of the numbers, so formally one may write the numbers as $a_0 \cdot 10^5 + a_1 \cdot 10^4 + a_2 \cdot 10^3 + a_3 \cdot 10^2 + a_4 \cdot 10 + a_5$). The first 5 digits are data digits, and a_5 is a check digit. The check digit is chosen in a way such that the weighted checksum

   $$S = \sum_{i=0}^{5} w_i \cdot a_i$$

   is divisible by 10, where the weights are $w_0 = 3$, $w_1 = 7$, $w_2 = 1$, $w_3 = 3$, $w_4 = 7$, $w_5 = 1$.

   (a) Determine if 123456 is a valid student ID number.

   (b) Find the missing digit X of the student ID number $9X9999$.

   (c) Find two student ID numbers $a_0a_1a_2a_3a_4a_5$ and $b_0b_1b_2b_3b_4b_5$ which differ in exactly 2 positions.

   (d) At most how many errors can be detected with this code, that is, find the largest t such that the code is t-error-detecting?

   (e) Could any other weights w_i have been used than 1, 3, and 7 to get the same minimum distance as the given code? If no, motivate. If yes, what weights?

   (f) Why are the given weights used in the particular order and not, for example, as $w_0 = 1$, $w_1 = 1$, $w_2 = 3$, $w_3 = 3$, $w_4 = 7$, $w_5 = 7$?

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2. (6p.) Error-correcting codes.

Consider a binary code with parity check matrix

$$H = \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 1 & 0 \end{bmatrix}.$$

(a) Find a generator matrix $\boldsymbol{G}$ for the code.

(b) What is the rate of the code?

(c) What is the minimum distance of the code?

(d) What should the decoder do when this code is used and the word 10011000 is received?

(e) What should the decoder do when this code is used and the word 10101010 is received?

(f) What should the decoder do when this code is used and the word 11000001 is received?

3. (6p.) Finite fields.

(a) Is the polynomial $2x^2 + 4 \in \mathrm{GF}(5)[x]$ irreducible?

(b) Is the polynomial $x^3 + x^2 + 1 \in \mathrm{GF}(2)[x]$ irreducible?

(c) Is there a finite field of order 729, GF(729)?

(d) Is there a nonzero element $t \in \mathrm{GF}(4)$ that satisfies $t + t = 0$?

(e) Determine $\phi(10)$, where $\phi(t)$ is the Euler totient function.

(f) Solve the equation $x^4 + 3x^3 + 2x + 1 = 0$, when $x \in \mathrm{GF}(5)$ and multiplication and addition is carried out in GF(5). Let the elements of the field be $\{0, 1, 2, 3, 4\}$.

4. (6p.) Convolutional codes. Draw a noncatastrophic rate-1/2 convolutional code. Give an example of how an input sequence is encoded using this encoder. Then change two bits of the encoded sequence and check whether Viterbi decoding can correct the errors (assuming that a binary symmetric channel, BSC, has been used for the transmission).