Aalto University School of Electrical Engineering
Department of Communications and Networking
ELEC-E7450 Performance Analysis, Spring 2016

Examination
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1. Consider a system of two parallel single-server queues with service rates μ_1 and μ_2. New jobs arrive according to a Poisson process with rate λ. At the arrival time, the arriving job is assigned to one of the queues depending on the state of the system described by vector $n = (n_1, n_2)$, where n_i denotes the number of jobs in queue i. The job assignment rule is so called *join-the-shortest-queue* (JSQ), i.e., the arriving job is assigned to queue $q(n) \in \{1, 2\}$ for which
$$q(n) = \arg\min\{n_1, n_2\}.$$
If $n_1 = n_2$, then the tie-breaking rule is uniformly random, i.e., each queue is chosen with the same probability 1/2. The jobs assigned to queue i have IID exponential service times with mean $1/\mu_i$. Let $N_i(t)$ denote the number of jobs in queue i at time t. The pair $(N_1(t), N_2(t))$ constitutes a two-dimensional Markov process. Draw its state transition diagram.

2. Consider an M/G/1-FIFO queue with two customer classes. Class-k customers ($k = 1, 2$) arrive according to an independent Poisson process with intensity λ_k (customers/sec). Service times for class-k customers are exponentially distributed with mean $1/\mu_k$ (sec), where $1/\mu_1 = 1$ sec and $1/\mu_2 = 3$ sec. Determine
   (a) the stability conditions for arrival rates λ_1 and λ_2;
   (b) the mean steady-state number of class-1 customers when $\lambda_1 = \lambda_2 = 1/5$?

3. Consider flow-level data traffic in a bottleneck link of an IP network. The traffic consists of elastic flows (such as file transfers using TCP), which arrive according to a Poisson process with rate λ (flows/sec). The flow sizes B_i are IID random variables with mean $E[B] = 30$ Mb, and the link capacity is $C = 100$ Mbps. Use the M/G/1-PS model to determine
   (a) the stability limit for the arrival rate λ;
   (b) the mean file transfer time for a randomly chosen flow when $\lambda = 3$ (flows/sec).

4. Consider the following model for packet traffic in a router. New packets arrive according to a Poisson process with intensity λ (packets/ms). All packets are first processed in a processor (possibly even multiple times). All processing times are independently and exponentially distributed with mean $1/\mu_0 = 1$ ms. After a packet is processed (or reprocessed), it is routed
   - back to the same processor queue (for reprocessing) with probability p_0,
   - to outgoing link 1 with probability p_1, and
   - to outgoing link 2 with probability p_2,

   where $p_0 = 1/2$, $p_1 = 1/3$, and $p_2 = 1/6$. The transmission times on link k are independently and exponentially distributed with mean $1/\mu_k$, where $1/\mu_1 = 4$ ms and $1/\mu_2 = 6$ ms. Determine
   (a) the stability limit for the arrival rate λ;
   (b) the mean time that a packet spends in the whole system when $1/\lambda = 3$ ms.

5. Consider a linear network with $J = 3$ links and $K = 4$ classes of flows. All links have the same capacity C (100 Mbps). The flows in classes $k = 1, 2, 3$ use a single link (i.e., link k), while the flows in class $k = 4$ use all three links. Let $n_k > 0$ denote the number of flows in class k, and assume that $n_1 = 1$, $n_2 = 2$, $n_3 = 3$, $n_4 = 4$. Determine the proportional fair (PF) inter-class allocations $\phi_k(n)$ for each class k.