PHYS-E0563 Fundamentals of Plasma Physics for Space and Fusion Applications Exam 12.5.2014

Answer to **FOUR** questions.

Plasma phenomena and general aspects

1. Give short (not more than 1/3 page for each) but complete answers to following questions:

   a) Give a function that is a solution to collisionless Boltzmann equation. (1p)

   b) Explain how the helical magnetic field of a tokamak is generated. (1p)

   c) What is meant by the plasma stability? (1p)

   d) What happens in the resonance of the cold plasma waves? (1p)

   e) What is meant by a quasi-neutral hybrid simulation model for space applications? (1p)

   f) Give at least one reason to use Hamiltonian mechanics over the non-Hamiltonian one. (1p)

Single particle dynamics

2. a) Starting from the charged particle Lagrange function

$$L = \frac{1}{2} m\dot{\boldsymbol{r}} \cdot \dot{\boldsymbol{r}} + q\dot{\boldsymbol{r}} \cdot \boldsymbol{A}(\boldsymbol{r}, t) - q\Phi(\boldsymbol{r}, t),$$

   derive the equation of motion for $\boldsymbol{r}$ using the Euler-Lagrange equation $\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\boldsymbol{r}}}\right) = \frac{\partial L}{\partial \boldsymbol{r}}$. (4p)

   b) The Euler-Lagrange equation $\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\boldsymbol{R}}}\right) = \frac{\partial L}{\partial \boldsymbol{R}}$ implies that for a given symmetry coordinate y in the Lagrangian (i.e. the Lagrangian does not depend on coordinate y), there is a conserved quantity $\frac{\partial L}{\partial \dot{y}}$. Derive a relation for the conserved guiding center quantity in case of a toroidally symmetric tokamak (neglecting the electric field), where the guiding center Lagrangian is given by

$$L = \left(q\boldsymbol{A} + mv_\parallel \hat{\boldsymbol{b}}\right) \cdot \dot{\boldsymbol{R}} - \frac{1}{2} mv_\parallel^2 - \mu B.$$

   Toroidally symmetric magnetic field is given by $\boldsymbol{B} = \boldsymbol{B}(R, z)$. Here the phase-space is composed of the toroidal coordinates $\boldsymbol{R} = (R, \phi, z)$ and velocity components $v_\parallel$ and μ. The conserved quantity found in this problem is called the toroidal canonical momentum. (2p)

Kinetic theory - collisions

3. a) Collision operator for species a, $C_a(f_a)$, should satisfy following equations

$$\int C_a(f_a)\, \mathrm{d}^3\boldsymbol{v} = 0 \tag{1}$$

   and

$$\sum_a \boldsymbol{F}_a = 0, \tag{2}$$

   where $\boldsymbol{F}_a$ is the force due to collisions, and it is defined by

$$\boldsymbol{F}_a = \int m_a \boldsymbol{v} C_a(f_a)\, \mathrm{d}^3\boldsymbol{v}.$$

   What is the physical meaning of equations (1) and (2)? (2p)

   b) In the exercises we showed that an approximation for the collision operator can be expressed as

$$C_a(f_a) = \sum_b \frac{c_{ab}}{m_a} \frac{\partial}{\partial \boldsymbol{v}} \cdot \int d\boldsymbol{v}' \left(\frac{\boldsymbol{I}}{u} - \frac{\boldsymbol{uu}}{u^3}\right) \cdot \left(\frac{1}{m_a} \frac{\partial f_a}{\partial \boldsymbol{v}} f_b(\boldsymbol{v}') - \frac{1}{m_b} \frac{\partial f_b}{\partial \boldsymbol{v}'} f_a(\boldsymbol{v})\right).$$

   Proove that either (1) or (2) (NOT BOTH) is satisfied for this collision operator. (3p)

   c) Can you list any other properties the collision operator should satisfy? (1p)

TURN

Collisional cold plasma waves

4. a) In the lectures, we derived the cold plasma dielectric tensor for a MHD fluid from the simple equations of motion acting on the fluid element. Your task is now to include a frictional force $F_s = -m_s \nu \mathbf{v}_{1s}$ in the equations of motion. Frictional force can exists when several fluids are considered, unlike in single fluid MHD. By doing this, show that the original collisionless calculation is valid, as long as we replace the original frequency ω by an imaginary frequency $\omega' = \omega + i\nu$. (3p)

   b) Solving the dispersion relation for an ordinary wave (O-wave), using ω', leads to a solution $\omega = \omega_r - \frac{\omega_p^2}{\omega_r^2}\frac{\nu}{2}i$ (here ω_r is real and we have assumed $\frac{\nu}{\omega} \ll 1$). Is the O-wave growing or damped due to collisions? (2p)

   c) Give a reason why plasma current density and electric field in a magnetized plasma are not aligned, i.e. we need a dielectric tensor rather than dielectric constant? (1p)

MHD stability

5. Consider a cylindrically symmetric plasma column ($\partial_z = 0$, $\partial_\theta = 0$; z is the direction of the cylinder axis) under equilibrium conditions, confined by a magnetic field. Verify that in cylindrical coordinates the hydromagnetic equilibrium is given by

$$\frac{dp(r)}{dr} = j_\theta(r) B_z(r) - j_z(r) B_\theta(r) . \text{(3p)}$$

Using Ampere's law, derive the equilibrium equation to the form

$$\frac{d}{dr}\left(p + \frac{B_\theta^2}{2\mu_0} + \frac{B_z^2}{2\mu_0}\right) = -\frac{B_\theta^2}{\mu_0 r} . \text{(3p)}$$