CHEM-E7155. Basics in production planning and control.
Exam, 3.10.2016

1. Answer shortly the following questions:

a) What is the essence of the process optimization approach employed by the operation research?
b) Give definition of the shadow prices in Linear Programming and explain what the shadow prices can be used for.
c) Please explain how dummy variables are used to fit a problem into the transportation linear programming formulation.
d) Which type of problems is easier to solve: liner programming or integer programming? Explain your answer.
e) Give definition of the Convex Programming. Provide an example of optimization problem, which is not a convex programming.
f) Please describe briefly the basic optimality criteria used in production scheduling.

2. Linear and nonlinear programming:

a) Linear programming

Solve the following problem using the simplex method

$$\max Z = 5x_1 + 4x_2 - x_3$$

$$x_1 + x_2 \leq 2$$
$$x_1 - x_3 \leq 1$$
$$x_1 + x_2 + x_3 \leq 2$$
$$x_1, x_2, x_3 \geq 0$$

b) Nonlinear programming

Consider the following nonlinear programming problem with an equality constraint:

$$\text{minimize } f(x_1, x_2, x_3) = 4x_1^2 + 5x_2^2 + x_3$$

subject to:

$$x_2 + 3x_3 = 6$$

Write down the Lagrange function and obtain the resulting system of equations. You are not required to solve the system of equations!

3. Formulate optimization mathematically based on the text description:

c) Formulate linear programming

A company plans to blend a new alloy of 40% tin, 35% zinc and 25% lead from several available alloys having the following properties:

Property	Alloy				
	1	2	3	4	5
tin, %	60	25	45	20	50
zinc, %	10	15	45	50	40
lead, %	30	60	10	30	10
COST ($/kg)	22	20	25	24	27

The objective is to determine the proportions of the alloys that should be blended to produce the new alloy at a minimum cost. Please formulate the problem as a linear programming task. Do not solve it

a) Formulate integer programming

An airline company is considering the purchase of new long, medium and short-range airplanes. The prices is 67 M$ for each long-range plane, 50 M$ for a medium-range plane and 35 M$ for a short-range plane. The funds available is 1500 M$. The estimated annual profit would be 4.2 M$ for a long-range, 3 M$ for a medium-range, and 2.3 M$ for a short-range planes. There is enough prepared pilots for 30 crews. The maintenance facilities are able to handle 40 short-range planes. However, a medium range plane is equivalent to 4/3 short range planes, and a long-range plane is equivalent to a 5/3 short-range planes in terms of their use of the maintenance facilities.

Please formulate the Integer Programming problem mathematically. Explain how you would reformulate it as a Binary Programming problem.

4. Transportation simplex:

Consider the (minimization) transportation problem below:

		Destination				
		1	**2**	**3**	**4**	**Supply**
Source	1	10	0	20	11	15
	2	12	7	9	20	25
	3	0	14	16	18	5
Demand		5	15	15	10	

Use the Northwest corner rule to obtain an initial BF solution. Starting with the initial BF solution apply the transportation simplex method to obtain an optimal solution.

5. Dynamic programming

The owner of a chain of three grocery stores has purchased five packages of a product. The estimated profitability of potential sales of the product differs among the three stores. Therefore, the owner wants to know how to allocate five packages to the three stores to maximize the expected profit. For administrative reasons, the owner does not wish to split packages between stores. The following table gives the estimated expected profit at each store when allocated various numbers of packages.

	Store		
Packages	**1**	**2**	**3**
0	0	0	0
1	5	6	4
2	9	11	9
3	14	15	13
4	17	19	18
5	21	22	20

Use dynamic programming to determine how many of the five packages should be assigned to each of the three stores to maximize the total expected profit.