ELEC-E8101 Digital and Optimal Control
Full exam 9. 5. 2016

- *Write the name of the course, your name, your study program, and student number to each answer sheet.*
- *There are five (5) problems and each one must be answered.*
- *No other literature except the Table of Formulas is allowed. A function calculator can be used.*
- *The table of formulas must be returned, if you have received it from the exam supervisor.*
- *Mark clearly FULL EXAM on the answer sheet.*

Each problem given a maximum of 6 points.

1. You are a *tutor* of students, who are facing an examination on computer control. One of them is desperately seeking information on the following concepts. Write short explanations to him. Note that he is a true ice-hockey fan and very busy watching the games of the on-going World championship tournament, so he does not want to read long explanations.

   - zero order hold
   - anti-alias filter
   - minimum-variance control
   - LQ control
   - coloured noise
   - ARMAX-model

2. Explain, what issues must be considered, when the sampling interval is chosen to a discrete-time controller.

3. Consider the closed-loop system representation

$$x(k+1)=\begin{bmatrix}0 & 1\\ -8 & 6\end{bmatrix}x(k),\quad x(0)=\begin{bmatrix}0\\ 10\end{bmatrix}$$

$$y(k)=\begin{bmatrix}1 & 1\end{bmatrix}x(k)$$

Calculate an analytical solution for $y(k)$ and by using it solve the values $y(0)$, $y(1)$ and $y(2)$. Hint: Use the z-transform.

4. Consider the following problem assuming that the optimization horizon (t_f) approaches infinity.

$$\dot{x}(t)=u(t),\quad x(0)=x_0$$

$$J(0)=\frac{1}{2}\int_0^{t_f}\left(x^2+u^2\right)dt$$

a. Solve the optimal control. What is the optimal cost?

b. Determine the closed loop equation and solve the optimal trajectory. Show that it is stable.

5. Consider the system with the transfer function

$$G(s) = \frac{Y(s)}{U(s)} = \frac{\alpha^2}{s^2 + \alpha^2}, \quad \alpha \text{ is constant}$$

a. Make a state-space representation of the system using the state variables $x_1 = y, x_2 = (1/\alpha)\dot{y}$

b. By assuming zero order hold and sampling interval h calculate an equivalent discrete-time state- space representation

c. Assume that the sampling interval is modified as $h_n = h + n \cdot (2\pi/\alpha), n = 0,1,2,\ldots$. What happens to the discretized equations? Explain?

These formulas can be used, if needed.

$$\dot{x} = Ax + Bu, \quad t \geq t_0$$

$$J(t_0) = \frac{1}{2}x^T(t_f)S(t_f)x(t_f) + \frac{1}{2}\int_{t_0}^{t_f}\left(x^T Q x + u^T R u\right)dt \qquad S(t_f) \geq 0, \quad Q \geq 0, \quad R > 0$$

$$-\dot{S}(t) = A^T S + SA - SBR^{-1}B^T S + Q, \quad t \leq t_f,$$

boundary condition $S(t_f)$

$$u^* = -Kx \quad K = R^{-1}B^T S$$

$$J^*(t_0) = \frac{1}{2}x^T(t_0)S(t_0)x(t_0)$$