ELEC-E8101 Digital and Optimal Control
Intermediate exam 26. 10. 2016

- *Write the name of the course, your name, your study program, and student number to each answer sheet.*
- *There are three (3) problems and each one must be answered.*
- *No other literature except the Table of Formulas (Digital Control) is allowed. A function calculator can be used.*
- *The table of formulas must be returned, if you have received it from the exam supervisor.*
- *Mark clearly INTERMEDIATE EXAM 1 on the answer sheet.*

Max 5 points / problem

1. Consider a process with the transfer function $G(s) = \frac{2}{s+3}$. Let us discretize it by assuming the zero-order hold and by using the sampling interval h.

a. Determine the discrete-time pulse transfer function and state-space representation. (3 p)

b. Assume now that the discrete-time process model (from part a) is controlled by a P-controller (negative feedback) with the gain K. For what values of K is the closed-loop system stable? (2 p)

2. Consider the model

$$\begin{cases} x(kh+h) = \begin{bmatrix} 1 & h \\ 0 & 1 \end{bmatrix} x(kh) + \begin{bmatrix} \frac{1}{2}h^2 \\ h \end{bmatrix} u(kh) \\ y(kh) = \begin{bmatrix} 1 & 0 \end{bmatrix} x(kh) \end{cases}$$

where h is the sampling interval.

a. Determine the pulse transfer function. (3 p)
b. Is the system i. reachable, ii. observable? Explain in words what your answers mean. (2 p)

3. Consider the continuous time system

$$\dot{x} = \begin{bmatrix} 0 & \omega \\ -\omega & 0 \end{bmatrix} x + \begin{bmatrix} 0 \\ \omega \end{bmatrix} u$$
$$y = \begin{bmatrix} 1 & 0 \end{bmatrix} x$$

where ω is a positive constant.

a. What kind of a dynamic behaviour does the system represent? (1 p)

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b. The system is discretized with the sampling interval h assuming zero order hold. The resulting state-space representation is

$$x(kh+h) = \begin{bmatrix} \cos \omega h & \sin \omega h \\ -\sin \omega h & \cos \omega h \end{bmatrix} x(kh) + \begin{bmatrix} 1-\cos \omega h \\ \sin \omega h \end{bmatrix} u(kh)$$
$$y(kh) = \begin{bmatrix} 1 & 0 \end{bmatrix} x(kh)$$

How are the poles of the original continuous time system and the discrete equivalent related? Show that the theoretical result is valid in this example case. (2 p)

c. Let the constant ω change as $\omega_{new} = \omega + n \cdot \dfrac{2\pi}{h}$, $n = 0, 1, 2, \ldots$ What happens in the discretized model? Explain. (2 p)

Hint:

$$\sin^2(x) + \cos^2(x) = 1$$
$$e^{\pm jx} = \cos(x) \pm j\sin(x)$$