ELEC-E8101 Digital and Optimal Control
Intermediate exam 1. 23.10.2015

- *Write the name of the course, your name, your study program, and student number to each answer sheet.*
- *There are three (3) problems and each one must be answered.*
- *No other literature except the Table of Formulas is allowed. A function calculator can be used.*
- *The table of formulas must be returned, if you have received it from the exam supervisor.*
- *Mark clearly INTERMEDIATE EXAM 1 on the answer sheet.*

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1. Consider the process of a double integrator $G(s) = \frac{1}{s^2}$. Let us discretize it by assuming the zero-order hold and by using the sampling interval h.

   a. Present the pulse transfer function and state-space representation of the discretized process.
   b. The continuous time process is controlled by a P controller with the gain K, which can have any real tuning value. Is it possible to choose K such that the closed loop is asymptotically stable? (Show the necessary calculations and explain).
   c. The discretized process is controlled by a P controller. Is it possible to choose K such that the closed loop is asymptotically stable? (Show the necessary calculations and explain).

2. Consider the simple motor model

$$\dot{x}(t) = \begin{bmatrix} -1 & 0 \\ 1 & 0 \end{bmatrix} x(t) + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u(t)$$
$$y(t) = \begin{bmatrix} 0 & 1 \end{bmatrix} x(t)$$

The model is discretized by assuming zero-order hold and using the sampling interval h.

a. Show that a discrete-time realization is

$$x(kh+h) = \begin{bmatrix} e^{-h} & 0 \\ 1-e^{-h} & 1 \end{bmatrix} x(kh) + \begin{bmatrix} 1-e^{-h} \\ h-1+e^{-h} \end{bmatrix} u(kh)$$
$$y(kh) = \begin{bmatrix} 0 & 1 \end{bmatrix} x(kh)$$

b. Is it possible to design a feedback control law to the discrete model such that the closed-loop poles are at $0.5 \pm j0.5$ (j is the imaginary unit)? Assume that the states are measurable. Is the closed loop stable then? Note: You have to give an analytical answer to the question. In other words you have to confirm your answer by calculations and/or theory. Simple yes/no is not enough.
c. Assume that the states cannot be measured. Show how to design a state observer and how it can be used in the state feedback control. (You do not have to present calculations, just show the necessary formulas and how you would do the design.)

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3. The pole pair of a continuous-time system are the zeros of the denominator of

$$G(s) = \frac{\omega^2}{s^2 + 2\varsigma\omega s + \omega^2}$$

where ω is the natural (angular) frequency and ς the damping coefficient. Let the poles be a. complex, b. real. Where in the complex plane do the poles map when the system is discretized by assuming zero order hold and using the sampling interval h?