## Support material permitted: attached collection of formulas.

Write on each and every page:
Code and name of the course
Name and student number
Department and date
Return this text together with your solutions
1.
a) Define the lumped system approximation, and show under which conditions it is valid. Give an explicit physical example.
b) One of the walls of a furnace is 5 m high, 8 m long, and 0.22 m thick. The thermal conductivities of the various materials used are $k_{A}=k_{F}=2 \mathrm{~W} / \mathrm{m}^{\circ} \mathrm{C}, k_{B}=8 \mathrm{~W} / \mathrm{m}^{\circ} \mathrm{C}, k_{C}=$ $20 \mathrm{~W} / \mathrm{m}^{\circ} \mathrm{C}, k_{D}=15 \mathrm{~W} / \mathrm{m}^{\circ} \mathrm{C}$ and $k_{E}=35 \mathrm{~W} / \mathrm{m}^{\circ} \mathrm{C}$. The left and right surfaces of the wall are maintained at uniform temperatures $300^{\circ} \mathrm{C}$ and $100^{\circ} \mathrm{C}$, respectively (see the figure below). Heat transfer through the wall is assumed to be one-dimensional and steady.
i) Compute the rate of heat transfer through the wall
ii) Show how differently you would compute the heat rate if the structure were a thick singlelayer in transient heat transfer (comment on the formulas and their validity conditions).


## 2.

a) Explain the driving forces of natural convection, and make a comparison with forced convection. How does the heat transfer coefficient typically differ in these two cases? Comment also on the Grashof number Gr, focusing in particular on its physical meaning.
b) Hot air at $80^{\circ} \mathrm{C}$ enters a thin, smooth $10-\mathrm{m}$-long attic duct of a house with cross section $15 \mathrm{~cm} \times 20 \mathrm{~cm}$ at $V_{m}=5 \mathrm{~m} / \mathrm{s}$. The duct surface is uniformly and approximately at $T_{s}=70^{\circ} \mathrm{C}$. Calculate:
i) the heat transfer coefficient
ii) the temperature of the air at outlet (recall that the mass flow rate is $\dot{m}=\left(\frac{d m}{d t}\right)=\cdots$ )
iii) the pressure drop in the duct


Hint: assume the mean bulk temperature for the air inside the duct as $T_{b}=75^{\circ} \mathrm{C}$, it is easy to estimate the air properties (you will then check if this is consistent with $T_{e}$ ).
Remember that the entry lengths in this case are $L_{h} \approx L_{t} \approx 10 D_{H}$.
3.
a) Consider page 18 in the Collection of formulas included. Define a blackbody and explain the physical meaning of the Planck's law. Why do we define the blackbody radiation function? How do you use the table at page 18 ?
b) In a hemispherical ice hockey hall, the ice rink is a circle with diameter $D=30 \mathrm{~m}$.
i) Find the heat transfer rate to the entire ceiling (surface 2) from a sector (surface 1) which is $1 / 4$ of the ice rink, see the figure.
ii) What is the maximal value admissible for $\epsilon_{2}$, if the ice hall design cannot tolerate a heat rate from the ceiling which is larger than $\dot{Q}_{21} \sim 8 \mathrm{~kW}$ ?


Data: $\epsilon_{1}=0.98, T_{1}=-5^{\circ} \mathrm{C}$ and $T_{2}=10^{\circ} \mathrm{C}$. Remember that the surface area of a sphere of radius $r$ is $4 \pi r^{2}$.
4.
a) Examine the psychrometric chart below. Discuss both its usage and the following definitions: dry bulb Temperature, wet bulb Temperature, dew point and relative humidity.


Figure 20.6 Psychromatric chart
b) The $A=6 \mathrm{~m}^{2}$ external wall of a house consists of the following layers: gypsum, 9.5 mm ; mineral wool, 20 cm ; concrete, 10 cm ; bricks, 100 mm . The room temperature is $T_{i}=20^{\circ} \mathrm{C}$, the outer temperature is $T_{o}=5^{\circ} \mathrm{C}$. The relative humidity is respectively $\phi_{i}=60 \%$ and $\phi_{o}=85 \%$.

Compute the maximum amount of water vapor which will diffuse through the wall in 24 hrs. Hint: the overall vapor resistance is analogous to thermal resistance.

## 5.

Consider numerical methods in heat transfer studies.
a) In steady state h.t., what is the physical principle which associates an equation to a volume element? Why do we use numerics instead of analytical methods?
b) Consider a large plane wall of thickness $L=0.4 \mathrm{~m}$, thermal conductivity $k=2.3 \mathrm{~W} / \mathrm{m}^{\circ} \mathrm{C}$, and surface area $A=20 \mathrm{~m}^{2}$. The left side of the wall is maintained at constant $T_{0}=80^{\circ} \mathrm{C}$. The right side instead loses heat by convection to the surrounding air at $T_{\infty}=15^{\circ} \mathrm{C}$, with heat transfer coefficient $h=24 \mathrm{~W} / \mathrm{m}^{2} \mathrm{C}$.
Assuming steady one-dimensional heat transfer, and a nodal spacing $\Delta x=10 \mathrm{~cm}$,
i) verify that the nodal temperatures are $T_{1}=66.9^{\circ} \mathrm{C}, T_{2}=53.8^{\circ} \mathrm{C}, T_{3}=40.7^{\circ} \mathrm{C}, T_{4}=27.6^{\circ} \mathrm{C}$ by solving the according finite difference equations.
ii) compute the heat transfer rate through the wall.


