1: 2: 3: 4: Extra: **Total: [/ 28]**

Aalto ME-C3100 Computer Graphics, Fall 2015

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Välikoe/midterm, 23.10.2015

Allowed: One two-sided A4 sheet of notes, calculators (also symbolic). Turn your notes in with your answers. Write your answers in either Finnish or English.

Name, student ID: ______________________________

1 Linear Algebra and Transformations [/ 10]

1.1 Linearity [/ 2]

What properties characterize a linear function (operator) $L(x)$, with $x \in \mathbb{R}^n$? Write them down in one or two equations.

1.2 Linear vs. Rigid [/ 2]

Name one transformation that is linear but not rigid (Euclidean), and one transform that is rigid but not linear.

1.3 Rotation Matrices [/ 2]

Rotation matrices are characterized by $\mathbf{R}^{\mathrm{T}}\mathbf{R} = \mathbf{I}$ and $\det(\mathbf{R}) = 1$. Show that the matrix $\mathbf{RS}$ is a rotation matrix whenever $\mathbf{R}$ and $\mathbf{S}$ are rotation matrices. (Remember the elementary properties of determinants.)

1.4 Homogeneous Coordinates [/ 4]

Consider the coordinate-by-coordinate sum of two homogeneous points in 2D (each has components x, y, w). What is the geometric interpretation of this sum when both points have $w = 1$? [**/ 2**]

What about differing nonzero ws? Remember projective equivalence $(wx, wy, w) \equiv (x, y, 1)$ when $w \neq 0$; it's useful to think of the points as $(x_1w_1,\ y_1w_1,\ w_1)$ and $(x_2w_2,\ y_2w_2,\ w_2)$ here. [**/ 2**]

2 Hierarchical Modeling [/ 4]

2.1 Forward and Inverse Kinematics for Articulated Characters

a) Briefly describe what is meant by forward and inverse kinematics ("suora ja käänteinen kinematiikka") in the context of articulated characters (such as human models). What is the role of joint angles ("nivelkulma") in each case? [**/ 2**]

Motion capture means transferring an animation from a real actor onto a computer model. Which one of the two (forward or inverse kinematics) better describes the process? Why? [**/ 2**]

3 Curves, Splines [/ 6]

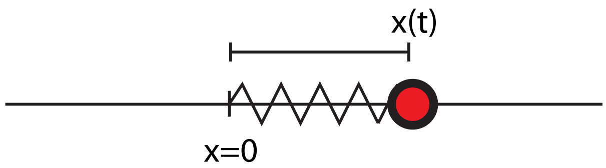
3.1 Continuity of B-Splines [/ 6]

A cubic B-spline is a piecewise cubic curve defined by a "sliding window" of 4 control points. Each set of 4 contiguous control points defines one segment of the spline. Consider the 2-segment spline given by 5 control points $\{P_1, \ldots, P_5\}$. The ith segment ($i = 1, 2$) is given by $P_i(t) = \sum_{j=1}^{4} B_j(t)\, P_{i+j-1}$, where the basis functions are defined as $B_1(t) = \frac{1}{6}(1-t)^3$, $B_2(t) = \frac{1}{6}(3t^3 - 6t^2 + 4)$, $B_3(t) = \frac{1}{6}(-3t^3 + 3t^2 + 3t + 1)$, $B_4(t) = \frac{1}{6}t^3$. Show that cubic B-splines are C^2 continuous, i.e., that $P_1(1) = P_2(0)$, and that also the tangents and second derivatives of the segments P_1 and P_2 agree at the join.

4 Physically-Based Animation [/ 8]

4.1 The Euler Method and Springs [/ 8]

Consider a unit mass (red circle) attached to the origin with a massless *damped spring* that is constrained to always lie along the x axis, and denote the position of the mass at time t with $x(t)$:



The motion of the spring is described by the differential equation

$$\frac{\mathrm{d}^2 x}{\mathrm{d}t^2} = -k(x-3) - l\frac{\mathrm{d}x}{\mathrm{d}t}$$

where the constants k, l are positive.

a) This is a second-order ordinary differential equation (ODE). Write down the corresponding first-order system of differential equations with variables x and $v = \mathrm{d}x/\mathrm{d}t$. *Yes, this is easy. If your result is complicated, you've gone astray!* [**/ 2**]

b) What is the rest length of the spring? What is the corresponding "rest position" of the free (non-attached) end when the spring is at rest? [**/ 2**]

c) Let the initial conditions be $x(0) = 1$ and $\frac{\mathrm{d}x}{\mathrm{d}t}(0) = 3$. What is the state of the system after one timestep of Euler integration with step size h? Give the values of x and v. [**/ 2**]

d) What is the physical significance/interpretation of the term $-l(\mathrm{d}x/\mathrm{d}t)$? Why can it be useful to use it in practice even if you wouldn't want the physical effect? [**/ 2**]

5 Extra Credit [/ 12]

Partial credit for the extra credit questions is only given if you make a technical mistake in solving an equation that is otherwise correctly derived. Continue your answers onto the next blank page if necessary.

5.1 Custom cubic spline [/ 6]

Derive the spline matrix for a cubic spline $\mathbf{P}(t)$ with the following properties:

1. it interpolates the first control point $\mathbf{P}_1$ at $t = 0$;
2. its tangent $\mathbf{P}'(t)$ at $t = 0$ matches $3 * (\mathbf{P}_2 - \mathbf{P}_1)$ like a cubic Bézier curve,
3. it interpolates the third control point, i.e., $\mathbf{P}(2/3) = \mathbf{P}_3$,
4. and its tangent at the end points away from $\mathbf{P}_1$ such that $\mathbf{P}'(1) = 3 * (\mathbf{P}_4 - \mathbf{P}_1)$. (Note that the curve generally won't interpolate $\mathbf{P}_4$.)

You will need to write out these constraints as a system of equations — it will turn out to be a linear system — and solve it to get the spline coefficients.

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5.2 Three Shears Make a 2D Rotation[/ 6]

Consider three two-dimensional shear operations applied in succession: one along x (by amount α) followed by another along y (by amount β), and finally re-applying the first shear along x:

$$\begin{pmatrix} 1 & \alpha \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ \beta & 1 \end{pmatrix} \begin{pmatrix} 1 & \alpha \\ 0 & 1 \end{pmatrix}. \tag{1}$$

Now set $\alpha = -\tan(\phi/2)$ and $\beta = \sin\phi$. Show that the product of the three matrices is a rotation by the angle ϕ. You may only consider $\phi < \pi/4$ to avoid singularities. You can do this by by writing out the product times its own transpose and noticing this is the identity (given our choices of α, β), or looking at the product itself and noticing its form. Obviously, you need to show the intermediate steps. *(Yes, this requires remembering your trigonometry.)*