

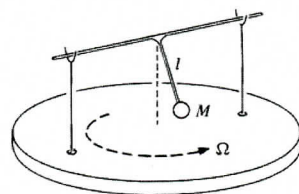
PHYS-E0413 Theoretical Mechanics
Midterm exam Wednesday 26.10.2016

1. Explain following concepts briefly (max 1p each)

- (a) Holonomic constraint
- (b) Virtual displacement
- (c) Action
- (d) Hamilton's principle
- (e) Hamiltonian and its relation to Lagrangian
- (f) Phase space

2. A pendulum of mass M and length l is attached to a horizontal rod that is in turn attached from both ends to a carousel rotating at an angular velocity

- (a) Write Lagrangian for the pendulum (3p).
- (b) By assuming a small oscillation angle derive the oscillation period of the pendulum. At what angular velocity will the motion cease to be harmonic (i.e oscillatory)? (2p)
- (c) What happens for faster rotations and what does it imply for your assumption about small oscillation angles? (1p)



3. A particle of mass m is moving without friction on the surface of a cylinder

$$x^2 + y^2 = R^2.$$

A (conservative) central force is acting on the particle, i.e. $\vec{F} = -k\vec{r}$ where \vec{r} is the position of the particle (in three-dimensions) and k is a positive constant.

a) Find the lagrangian and Lagrange's equations of motion in the cylindrical coordinates. (Remember that in cylindrical coordinates position is given by (r, ϕ, z) , where $r = \sqrt{x^2 + y^2}$). (3p.)

b) Determine the cyclic coordinates and the corresponding constants of the motion. (2p.)

c) Determine the cyclic coordinates and the corresponding constants of the motion if the force is only in xy -plane so that $\vec{F} = -k(x, y)$. (1p)

4. Particle of mass m moves in (x, y) plane under the influence of gravity $V = mgy$. There is a constraint $y = f(x)$.

a) Write the Lagrangian for the system and compute the generalized momentum and hamiltonian. (3p)

b) Write down Hamilton's equations (3p)

Remember to answer in english unless you have a special permission to use some other language. Write your name, student number, study program, course code, and the date in all your papers. Use of calculators is forbidden.