ELEC-C1320 – Robotiikka, Exam 11.1.2016

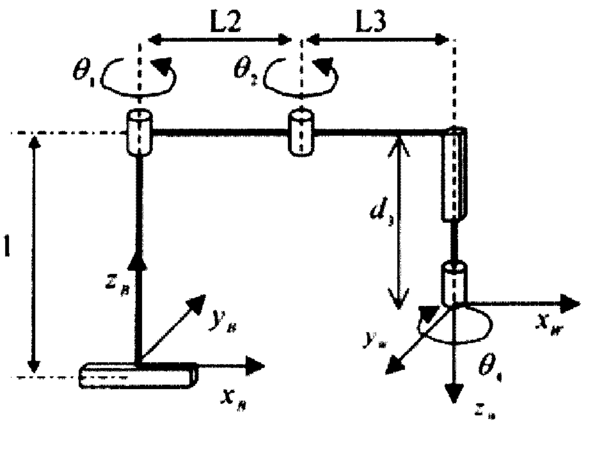
<u>It is allowed</u> to use a calculator and a book of mathematical equations (e.g. MAOL) in the exam.

1. The rotation matrix $\boldsymbol{R}$ describes the orientation of a coordinate frame with respect to the world frame: $\boldsymbol{R} = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & -1 & 0 \end{bmatrix}$

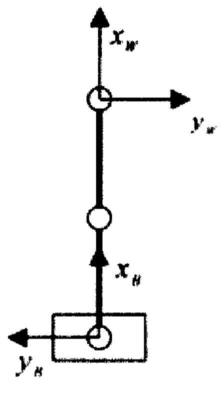
   a) The x-axis of the coordinate frame is parallel to the: a) world frame negative x-axis, b) negative y-axis, c) y-axis or d) negative z-axis ? (3 points)
   b) The y-axis of the coordinate frame is parallel to the: a) world frame negative x-axis, b) y-axis, c) negative y-axis or d) negative z-axis ? (3 points)
   c) The z-axis of the coordinate frame is parallel to the: a) world frame x-axis, b) y-axis, c) negative y-axis or d) z-axis ? (3 points)

2. In the figure below the kinematic structure of a four degree-of-freedom SCARA robot is shown. The first two joints are rotational (shoulder, θ_1, and elbow, θ_2, joints move the arm on a plane), then a prismatic joint, d_3 follows, which moves the tool up and down. And finally, in the kinematic chain, a rotational joint, θ_4 adjusts the orientation of the tool, for example, to grasp objects, which are laying on a pallet, oriented parallel to the xy-plane of the B-frame. In the figure, the manipulator is shown in its home/zero position (i.e. when all the joint control variables are zero, the upper arm is oriented horizontally above the x_B-axis and x_W is codirectional with x_b, y_W with $-y_b$ and z_W with $-z_b$).

   Number and mark in the figure the link-frames required for constructing the forward kinematic transformation of the manipulator for describing the wrist frame (W) with respect to the base frame (B). Also give in a table the link parameters and variables (i.e. Denavit-Hartenberg parameters). It is your choice to use either the Standard or Modified DH-parameter convention. (17 points)



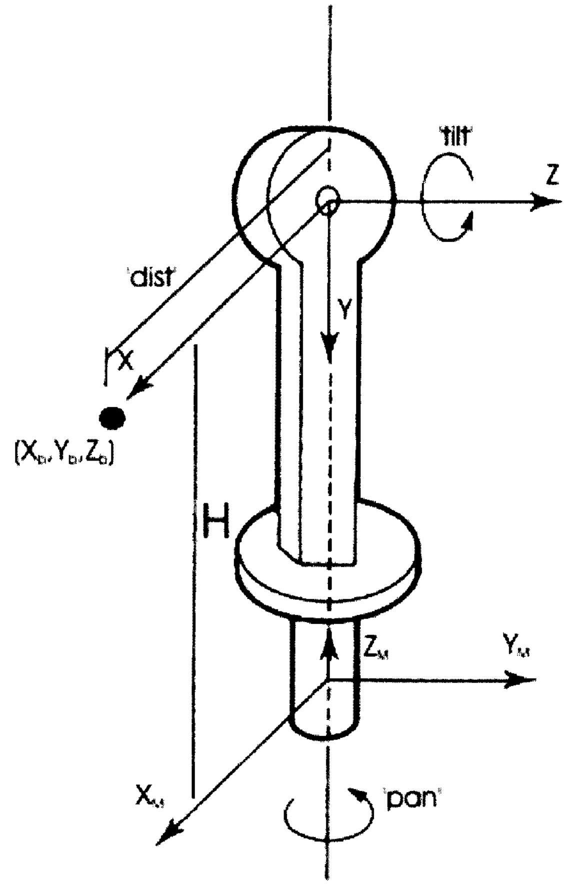
sivusta/ side view



ylhäältä/ top view

3. Solve the inverse kinematics of the pan/tilt mechanism pointing a laser range finder, which is shown in the image below. In other words, determine the equations:

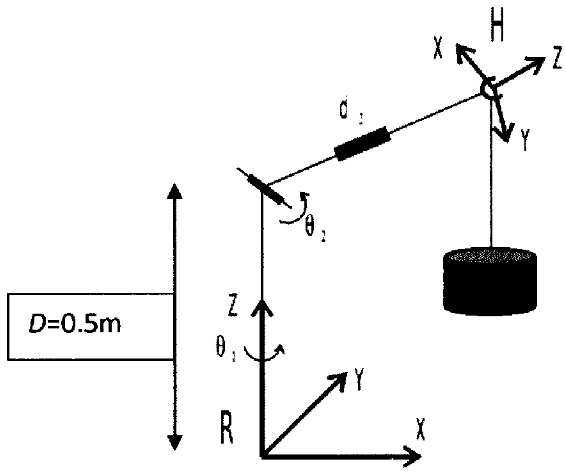
   dist = $f(x_b,y_b,z_b)$, pan = $f(x_b,y_b,z_b)$, tilt = $f(x_b,y_b,z_b)$ where x_b,y_b,z_b are the coordinates of the beam hit point expressed with respect to the M-frame. "H" is the height of axis of the vertical beam aiming mechanism (tilt) of the pan/tilt-pointing system above the origin of the M-frame (12 points)



4. The task is to form the manipulator Jacobian matrix for the RRP-robot shown below.(12 points)
The Jacobian matrix $\boldsymbol{J}$ describes the relationship between the linear velocity of the origin of the H-frame and velocities of the robot joints: $\begin{bmatrix} v_x \\ v_y \\ v_z \end{bmatrix} = \boldsymbol{J} \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \\ \dot{d}_3 \end{bmatrix}$

The forward kinematics solution of the robot in the form of manipulator arm matrix is

$$ {}^{R}T_{H} = \begin{bmatrix} -c\theta_1 s\theta_2 & s\theta_1 & c\theta_1 c\theta_2 & c\theta_1 c\theta_2 d_3 \\ -s\theta_1 s\theta_2 & -c\theta_1 & s\theta_1 c\theta_2 & s\theta_1 c\theta_2 d_3 \\ c\theta_2 & 0 & s\theta_2 & s\theta_2 d_3 + D \\ 0 & 0 & 0 & 1 \end{bmatrix} $$



5. A weight of 3kg is fixed to the tip of last link (at the location of the origin of the H-frame) of the three-link manipulator, introduced in problem 4 and shown in the figure above. The task is to <u>calculate torques and forces affecting joints 1, 2 and 3 (due to gravity) in two different configurations of the manipulator arm</u>. To solve the problem here **<u>you must utilize the Jacobian matrix</u>** formed in problem 4. The joint configurations to be considered are:

   a) θ_1=0.0°, θ_2=0.0°, d_3=0.5m (*the total length of the upper link is described by d_3*) (8 points)

   b) θ_1=0.0°, θ_2=90.0°,d_3=0.5m (*the total length of the upper link is described by d_3*) (7 points)

   The links itself are assumed to be weightless.
   The gravitational acceleration vector is pointing in the direction of negative Z_R-axis and its value is 9.81 m/s².

ELEC-C1320 Robotiikka - Equations

Link parameters and the corresponding elementary transformations as well as the symbolic form of the link matrix according to the **standard** Denavit and Hartenberg parameter convention:

$$^{j-1}\boldsymbol{A}_j(\theta_j, d_j, a_j, \alpha_j) = \boldsymbol{T}_{Rz}(\theta_j)\boldsymbol{T}_z(d_j)\boldsymbol{T}_x(a_j)\boldsymbol{T}_{Rx}(\alpha_j)$$

$$^{j-1}\boldsymbol{A}_j = \begin{pmatrix} \cos\theta_j & -\sin\theta_j\cos\alpha_j & \sin\theta_j\sin\alpha_j & a_j\cos\theta_j \\ \sin\theta_j & \cos\theta_j\cos\alpha_j & -\cos\theta_j\sin\alpha_j & a_j\sin\theta_i \\ 0 & \sin\alpha_j & \cos\alpha_j & d_j \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Link parameters and the corresponding elementary transformations as well as the symbolic form of the link matrix according to the **modified** Denavit and Hartenberg parameter convention:

$$^{j-1}\boldsymbol{A}_j = \boldsymbol{R}_x(\alpha_{j-1})\boldsymbol{T}_x(a_{j-1})\boldsymbol{R}_z(\theta_j)\boldsymbol{T}_z(d_j)$$

$$^{j-1}\boldsymbol{A}_j = \begin{bmatrix} c\theta_j & -s\theta_j & 0 & a_{j-1} \\ s\theta_j c\alpha_{j-1} & c\theta_j c\alpha_{j-1} & -s\alpha_{j-1} & -s\alpha_{j-1}d_j \\ s\theta_j s\alpha_{j-1} & c\theta_j s\alpha_{j-1} & c\alpha_{j-1} & c\alpha_{j-1}d_j \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Elementary rotation transformations (i.e. rotations about principal axis by Θ):

$$\boldsymbol{R}_x(\theta) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{pmatrix}$$

$$\boldsymbol{R}_y(\theta) = \begin{pmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{pmatrix}$$

$$\boldsymbol{R}_z(\theta) = \begin{pmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Inverse of a 4x4 transformation matrix:

$$\boldsymbol{T}^{-1} = \begin{pmatrix} \boldsymbol{R} & \boldsymbol{t} \\ \boldsymbol{0}_{1\times3} & 1 \end{pmatrix}^{-1} = \begin{pmatrix} \boldsymbol{R}^T & -\boldsymbol{R}^T\boldsymbol{t} \\ \boldsymbol{0}_{1\times3} & 1 \end{pmatrix} \qquad (2.21)$$

Derivation of trigonometric functions:

Dsinx = cosx

Dcosx = -sinx

Definition of (manipulator) Jacobian matrix:

If $\boldsymbol{y} = F(\boldsymbol{x})$ and $\boldsymbol{x} \in \mathbb{R}^n$ and $\boldsymbol{y} \in \mathbb{R}^m$ then the Jacobian is the $m \times n$ matrix

$$J = \frac{\partial F}{\partial \boldsymbol{x}} = \begin{pmatrix} \frac{\partial y_1}{\partial x_1} & \cdots & \frac{\partial y_1}{\partial x_n} \\ \vdots & \ddots & \vdots \\ \frac{\partial y_m}{\partial x_1} & \cdots & \frac{\partial y_m}{\partial x_n} \end{pmatrix}$$

Jacobian transpose transforms a wrench applied at the end-effector, ${}^0\boldsymbol{g}$ to torques and forces experienced at the joints $\boldsymbol{Q}$:

$$\boldsymbol{Q} = {}^0J(q)^T\, {}^0\boldsymbol{g}$$