**Course: ELEC-E8103 Modelling, Estimation, and Dynamic Systems**

# EXAMINATION 22.10.2015

**The examination will start from 22.10.2015 12:00 to 21:00. Please submit your solutions to the exam section of the course page in mycourses.aalto.fi before the deadline. The exam will be carried out in student's own computer at home. The exam should be done individually, and NO discussion is allowed.**

**Important Notification: you should only solve 3 questions based on the rules below.**

If your student number ends with 0, 1, 2, or 3, you should answer questions 1, 4, and 5.

If your student number ends with 4, 5, or 6, you should answer questions 2, 4, and 5.

If your student number ends with 7, 8, 9, or alphabetical letters, you should answer questions 3, 4, and 5.

**Submission of the exam papers**

You should submit your solutions to the Exam section of the Aalto University Mycourses page: https://mycourses.aalto.fi/course/view.php?id=5231&section=4. Your submission should include a single zip file named as "surname_studentNumber_Exam.zip", consisting of a pdf file named "surname_studentNumber_Exam.pdf", and the following MATLAB files: "problem1.m", "problem1.sym", "parameters.m", "problem2.m", and "sys01.sid", "sys02.sid", "sys03.sid", "sys04.sid", and "sys05.sid" for problem 3.

The hard deadline for submission of the exam solutions is 22.10.2015 at 21:00.

**Hints**

The physical laws in the exam are already discussed in the exercise sessions of the course, especially sessions 2 and 6.

## Exam Questions:

1. **(17 points)**
   **A mechanical system is shown in Fig. 1, where $u_1$ and $u_2$ (forces) are the inputs and $x_1$ and $x_2$ (displacements) are the outputs.**

   a) Derive the dynamic model of the system in the form of differential equations.

   (3 points)

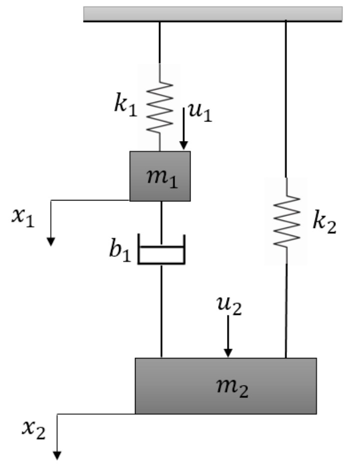


Figure 1 – Mass-Spring-Damper system

   b) Consider the given mechanical system for the following system parameters.

   $m_1$ = 20 kg

   $m_2$ = 70 kg

   $k_1$ = 800 N/m

   $k_2$ = 500 N/m

   $b_1$ = 30 N.s/m

   Assume the system is in equilibrium at t = 0.

   The input forces, $u_1$ and $u_2$, are sinusoidal functions with the following equations (in Newtons).

$$\begin{cases} u_1 = 2\sin(20\pi t + \frac{\pi}{2}) \\ u_2 = \sin(50\pi t) \end{cases}$$

   Write a MATLAB script named "problem1.m". Start from the differential equations derived in part "a". Simulate the system for the given functions and parameters. Plot the outputs of the system ($x_1$ and $x_2$) for the first 10 seconds of motion.

   (8 points)

c) Simulate the system in Simulink. Save the Simulink model as "problem1.sym". Use the inputs given in part "b". Plot the inputs ($u_1$ and $u_2$) and outputs ($x_1$ and $x_2$) of the system for the first 10 seconds of excitation.

You should include all the used parameters in the Simulink model in a separate file. After executing this file, all needed parameters should be stored to MATLAB workspace. Save the mentioned script -including the parameters for your simulation- as "parameters.m".

(6 points)

**2. (17 points)**

**A Resistor-Inductor-Capacitor system is shown in Fig. 2.**

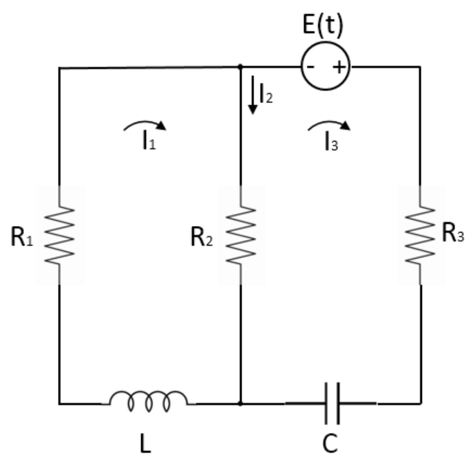


Figure 2 – RLC circuit

a) Derive the dynamic model of the system in the form of differential equations. (Hint: assume q (charge in capacitor) and $I_1$ as state variables)

(3 points)

b) Consider the given RLC circuit for the following input voltage function (E(t)).

$$E(t) = \sin(t)$$

The system parameters are as follows:

$R_1 = 4\ \Omega$

$R_2 = 3\ \Omega$

$R_3 = 2\ \Omega$

L = 1.6 H

C = 0.25 farads

$I_1(0)$ = 15 amps; Current in the left loop at t = 0

Q(0) = 2 amps.s; Charge in the capacitor at t = 0

Write a MATLAB script named “problem1.m”. Start from the differential equations derived in part “a”. Simulate the system for the given function and parameters. Plot the outputs of the system ($I_1$ and q) for the first 10 seconds of excitation.

(8 points)

c) Simulate the system in Simulink. Save the Simulink model as “problem1.sym”. Plot the input (E) and outputs ($I_1$ and q) of the system for the first 10 seconds of excitation.

You should include all the used parameters in the Simulink model in a separate file. After executing this file, all needed parameters should be stored to MATLAB workspace. Save the mentioned script -including the parameters for your simulation- as “parameters.m”.

(6 points)

**3. (17 points)**

**A magnetic suspension system is shown in Fig. 3. The RL circuit and the magnetic core are fixed and the magnetic ball is suspended. Gravity is downwards in the figure. Magnetic ball is energized and controlled by input voltage V. The control objective is to keep the distance of the magnetic core and the magnetic ball at a desired safe value.**

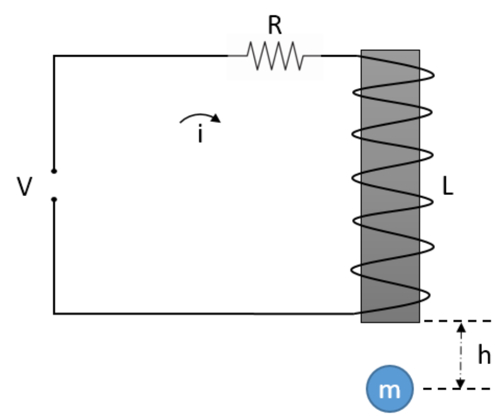


Figure 3 – Magnetic suspension system

Consider the given suspension system for the following magnetic force.

$$F(h, i) = k\frac{i^2}{h}$$

a) Derive the dynamic model of the system in the form of differential equations.

(1 point)

b) Consider the given suspension system for the following system parameters.

m = 0.05 kg

k = 0.0001 N/m

L = 0.01 H

R = 1 Ω

g = 9.81 m/s$^2$

Considering the voltage (V) as the input and distance (h) as the output, calculate the current (i) to keep the ball in equilibrium at h = 0.01 m.

(1 point)

c) Linearize the system around the equilibrium point calculated in part b.

(2 points)

d) Write a MATLAB script called "problem1.m". Obtain the state space representation of the system around the equilibrium point for the given function and parameters. Calculate the transfer function of the system.

(7 points)

e) Starting from the differential equations derived in part "a", simulate the system in Simulink. Save the Simulink model as "problem1.sym". Assume the input (V) with the following equation.

$$V = 0.1\sin(2\pi t)$$

Plot the input (V) and output (h) of the system for the first 10 seconds of excitation.

You should include all the used parameters in the Simulink model in a separate file. After executing this file, all needed parameters should be stored to MATLAB workspace. Save the mentioned script -including the parameters for your simulation- as "parameters.m".

(6 points)

**4. (8 points)**
**Load the data file "data4.mat". Consider x and y as training input and output, and xv and yv as validation input and output.**

Write a script called "problem2.m". The script should perform the following tasks:

a) Draw the scatter plot of (x,y) data. Estimate the parameters $k_i$ in the following model using the closed-form solution for the given data. (You are not allowed to use MATLAB curve fitting toolbox for this question)

$$y = k_1 x^{k_2}$$

(3 points)

b) Plot the estimated function for x ϵ [1,15] on the same scatter plot of data together with the validation data. Calculate SSE and $R^2$ for the training data. Calculate SSE for the validation data.

(2 points)

c) Estimate a polynomial model for the given data using MATLAB polynomial estimation functions, “polyfit” and “polyval”. What polynomial order do you choose for the given data? Why?

(3 points)

**5. (25 points)**
**Copy the files “runExam.m”, and “modeldata.mat” in your MATLAB current folder. Run the “runExam.m” script.**

The following text should appear in your command window.

```
>> runExam

Please Enter the numeric part of your student number!:
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Now, you should type your student number and press *Enter*. If your student number ends with an alphabetical letter, you should just type the numerical part of your student number, e.g. if your student number is 12345W, you should type 12345.

Then your data will be stored to MATLAB workspace. Data has been collected from five different dynamic systems. Input-output datasets are ($u_1$, $y_1$), ($u_2$, $y_2$), ($u_3$, $y_3$), ($u_4$, $y_4$), and ($u_5$, $y_5$).

Identify polynomial models of the mentioned systems using MATLAB System Identification Toolbox. You should explicitly select a model as your final answer for each input-output dataset.

You should explain the identification path in your final document. The identification path includes the following issues:

- What were the reason(s) for rejection of a specific model structure in your path?
- What were the reason(s) for selection of a specific model structure in your path?
- What are the alternative model(s) for the data if you think there are any?

You should validate your answers to the above-mentioned questions using required plots. You should include all validation test results for your final selected model for each system in your document.

Save the final identification session for each dynamic system as “sys01.sid” for ($u_1$ , $y_1$), “sys02.sid” for ($u_2$ , $y_2$), “sys03.sid” for ($u_3$ , $y_3$), “sys04.sid” for ($u_4$ , $y_4$), and “sys05.sid” for ($u_5$ , $y_5$).

The points will be given based on the resulted model structures and their orders, as well as the justification of the identification path.

(5 points for identification of each system, 25 points in total)