
Course: ELEC-E8103 Modelling, Estimation, and Dynamic Systems

EXAMINATION 27.10.2016

The examination will start from 27.10.2016 09:00 to 18:00. Please submit your solutions to the exam section of the course page in mycourses.aalto.fi before the deadline. The exam will be carried out online. The exam should be done individually, and **NO discussion is allowed**.

Important Notification: you should only solve 3 questions based on the rules below.

If your student number ends with 0, 1, 2, or 3, you should answer questions 1, 4, and 5.

If your student number ends with 4, 5, or 6, you should answer questions 2, 4, and 5.

If your student number ends with 7, 8, 9, or alphabetical letters, you should answer questions 3, 4, and 5.

Submission of the exam papers

You should submit your solutions to the **Exam** section of the Mycourses page: <https://mycourses.aalto.fi/mod/assign/view.php?id=160995>. Your submission should include a single zip file named as “surname_studentNumber_Exam.zip”, consisting of a pdf file named “surname_studentNumber_Exam.pdf”, and the following MATLAB files: “problem1.m”, “rhs.m”, “problem1.mdl”, “parameters.m”, “problem2.m”, and “sys01.sid”, “sys02.sid”, and “sys03.sid” for problem 5.

The hard deadline for submission of the exam solutions is 27.10.2016 at 18:00.

Hints

The physical laws in the exam are already discussed in the lectures of the course, especially sessions 1, 2, and 4.

1. (20 points)

The electromechanical system shown in Fig. 1 represents a simplified model of a capacitor microphone. The system consists in part of a parallel plate capacitor connected into an electric circuit. Capacitor plate “a” is rigidly fastened to the microphone frame. Sound waves pass through the mouthpiece and exert a force $f_s(t)$ on plate b, which has mass m and is connected to the frame by a set of springs and dampers. The capacitance C is a function of the distance between the plates, as follows:

$$C(x) = \frac{\epsilon A}{x}$$

Where ϵ is the dielectric constant of the material between the plates and A is the surface area of the plates. In this particular example, we assume vacuum between the plates. The charge q and the voltage e across the plates are related by the following equation:

$$q = C(x)e$$

The electric field in turn produces the following force f_e on the movable plate which attracts the movable plate towards the fixed plate because of opposite electric charges on the plates.

$$f_e = \frac{q^2}{2\epsilon A}$$

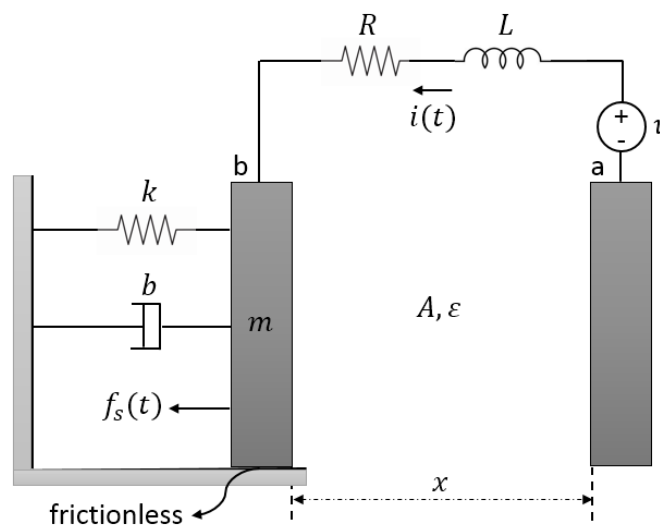


Figure 1 – Simplified model for capacitor microphone

k and b represent the spring and damper coefficients of the system, respectively. R and L represent the resistance and inductance of the electric circuit, respectively. v is the DC supply voltage of the electric circuit. Designers are interested in the current $i(t)$ which can be used in analyzing the exerted sound waves.

- a) Phase 1: Based on the description of the system, structure the problem. You are supposed to answer these sort of questions: What signal(s) are the output(s) of the system? What signal(s) are the input(s) of the system? What are the constants of the system? What are the internal time-varying variables of the system?

(1 point)

- b) Phase 2: Set up the basic equations of the system. You are supposed to derive the underlying differential equations of the system motion using first principles. What are the states of the system?

(5 points)

- c) In this part, you should solve the differential equations using MATLAB ode solvers. First, write a MATLAB script named “problem1.m”. Initialization and plotting should be done in this script. “problem1.m” should also call a function named “rhs.m” where the right-hand side of the equations are defined. Plot the response of the system (including all of the states) to the following input exerted force ($f_s(t)$) **for 2 seconds. Include the plots in your report.**

$$f_s(t) = 0.001\sin(100t)$$

The system parameters are as follows:

$$m = 0.01 \text{ kg}$$

$$k = 10 \text{ N/m}$$

$$b = 10 \text{ N.s/m}$$

$$\epsilon = 8.854 \times 10^{-12} \text{ F} \cdot \text{m}^{-1} \text{ (Assume vacuum)}$$

$$A = 2 \times 10^{-2} \text{ m}^2$$

$$R = 2 \times 10^6 \text{ } \Omega$$

$$L = 10^6 \text{ H}$$

$$v = 12 \text{ volts}$$

$$i(0) = 0 \text{ amps ; Current in the left loop at } t = 0$$

$$q(0) = 0 \text{ amps.s ; Charge in the capacitor at } t = 0$$

$$x(0) = 0.005 \text{ m ; Position of the plate at } t = 0$$

$$\dot{x}(0) = 0 \text{ m/s ; Velocity of the plate at } t = 0$$

Simulation time: [0, 2] seconds

(Important notification: This system is unstable. To stabilize the system, a mechanical lock on the plate is needed which is equivalent to a saturation block in Simulink. To simplify the question, we neglected the mechanical lock and you do not need to model any lock. However, in the aforementioned simulation time, your system will not go to infinity, but if you increase the simulation time, it will blow-up. Therefore, **make sure that you simulate the system only for 2 seconds)**

(7 points)

- d) Simulate the system in Simulink. Save the Simulink model as “problem1.mdl”. Plot the response of the system (states) for the given force in part c. **Include the plots in your report.** You should include all the used constants in the Simulink model in a separate file. After executing this file, all needed constants should be stored to MATLAB workspace. Save the mentioned script -including the constants for your simulation- as “constants.m”.

(7 points)

2. (20 points)

A magnetic suspension system is shown in Fig. 2. The magnetic coil is fixed and the ferromagnetic bar with mass m is suspended. Magnet is energized and controlled by current $i(t)$ and the objective is to predict $x(t)$, the distance of the magnet from the bar. The positive direction is assumed downwards. The bar is connected to a fixed frame by a set of springs and dampers. k and b are the spring and damper coefficients of the suspension system, respectively. Consider the magnetic force as a function of x and i with the following form.

$$F(z, i) = p \left(\frac{i}{z + \epsilon_0} \right)^2$$

Where $p = 1.5 \times 10^{-5}$, and $\epsilon_0 = 0.5 \text{ mm}$.

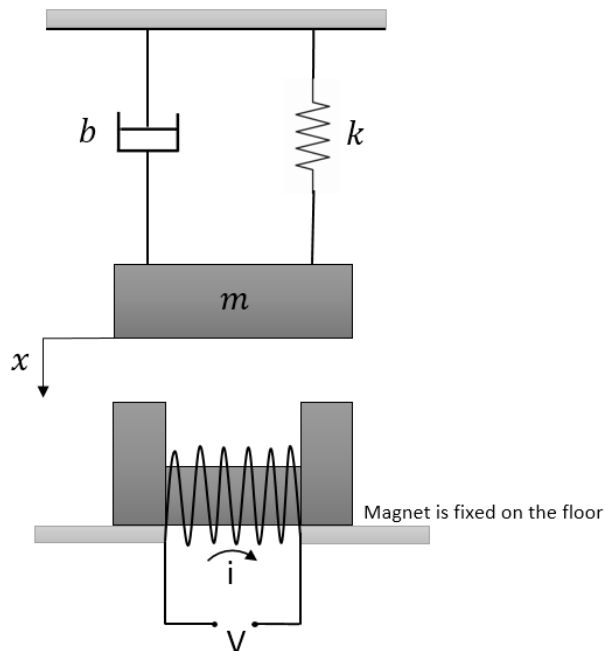


Figure 2 – Suspension system

- a) Phase 1: Based on the description of the system, structure the problem. You are supposed to answer these sort of questions: What signal(s) are the output(s) of the system? What signal(s) are the input(s) of the system? What are the constants of the system? What are the internal time-varying variables of the system?

(1 point)

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- b) Phase 2: Set up the basic equations of the system. You are supposed to derive the underlying differential equations of the system motion using first principles. What are the states of the system?

(5 points)

- c) In this part, you should solve the differential equations using MATLAB ode solvers. First, write a MATLAB script named “problem1.m”. Initialization and plotting should be done in this script. “problem1.m” should also call a function named “rhs.m” where the right-hand side of the equations are defined. Plot the response of the system (including all of the states) to the following input current ($i(t)$) **for 10 seconds. Include the plots in your report.**

$$i(t) = 10\sin(10t)$$

The system parameters are as follows:

$$m = 0.1 \text{ kg}$$

$$k = 10 \text{ N/m}$$

$$b = 10 \text{ N.s/m}$$

$$x(0) = 0.1 \text{ m ; Position of the bar at } t = 0$$

$$\dot{x}(0) = 0 \text{ m/s ; Velocity of the bar at } t = 0$$

Simulation time: [0,10] seconds

(7 points)

- d) Simulate the system in Simulink. Save the Simulink model as “problem1.mdl”. Plot the response of the system (states) for the given force in part c. **Include the plots in your report.** You should include all the used constants in the Simulink model in a separate file. After executing this file, all needed constants should be stored to MATLAB workspace. Save the mentioned script -including the constants for your simulation- as “constants.m”.
(7 points)

3. (20 points)

A mechanical system is shown in Fig. 3. An object m_2 is placed inside a box m_1 . The friction between the box and the object is neglected. A force f is applied to the box. The coupling between the box and the object is simplified by two sets of springs and dampers denoted by k_1 , b_1 , k_2 , and b_2 . The box has four wheels which exert rolling resistance to the box. The total rolling resistance applied to mass m_1 can be calculated as below. Rolling resistance is only applied to the box.

$$F_R = \mu(m_1 + m_2)g\dot{x}_1$$

We are interested in the positions and velocities of the box m_1 and the object m_2 resulted by the force f .

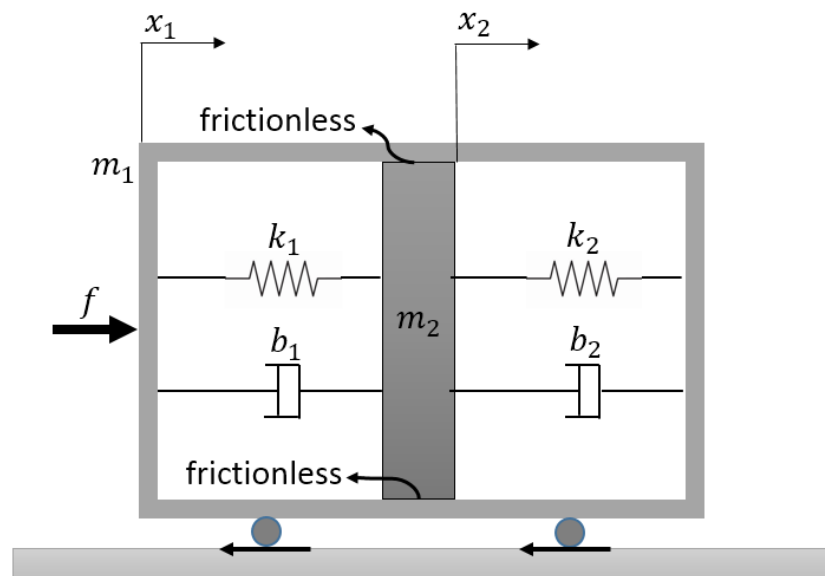


Figure 3 – Mechanical system

- a) Phase 1: Based on the description of the system, structure the problem. You are supposed to answer these sort of questions: What signal(s) are the output(s) of the system? What signal(s) are the input(s) of the system? What are the constants of the system? What are the internal time-varying variables of the system?

(1 point)

- b) Phase 2: Set up the basic equations of the system. You are supposed to derive the underlying differential equations of the system motion using first principles. What are the states of the system?

(5 points)

- c) In this part, you should solve the differential equations using MATLAB ode solvers. First, write a MATLAB script named “problem1.m”. Initialization and plotting should be done in this script. “problem1.m” should also call a function named “rhs.m” where the right-hand side of the equations are defined. Plot the response of the system (including all of the states) to the following input force ($f(t)$) **for 10 seconds. Include the plots in your report.**

$$f(t) = 0.1\sin(10t)$$

The system parameters are as follows:

$$m_1 = 4 \text{ kg}$$

$$m_2 = 2 \text{ kg}$$

$$k_1 = 1 \text{ N/m}$$

$$k_2 = 3 \text{ N/m}$$

$$b_1 = 1 \text{ N.s/m}$$

$$b_2 = 4 \text{ N.s/m}$$

$$\mu = 0.02$$

$$g = 9.8 \text{ m/s}^2$$

$$x_1(0) = 0 \text{ m ; Position of the box at } t = 0$$

$$\dot{x}_1(0) = 0 \text{ m/s ; Velocity of the plate at } t = 0$$

$$x_2(0) = 0 \text{ m ; Position of the object at } t = 0$$

$$\dot{x}_2(0) = 0 \text{ m/s ; Velocity of the object at } t = 0$$

Simulation time: [0,10] seconds

(7 points)

- d) Simulate the system in Simulink. Save the Simulink model as “problem1.mdl”. Plot the response of the system (states) for the given force in part c. **Include the plots in your report.** You should include all the used constants in the Simulink model in a separate file. After executing this file, all needed constants should be stored to MATLAB workspace. Save the mentioned script -including the constants for your simulation- as “constants.m”.
(7 points)

4. (9 points)

Load the data file “data2.mat”. Consider xt and yt as training input and output, and xv and yv as validation input and output.

Write a script called “problem2.m”. The script should perform the following tasks:

- a) Draw the scatter plot of (xt, yt) data. Estimate the parameters k_i in the following model using closed-form solution for the given data. **Write the estimated parameters in your report.** (You are not allowed to use MATLAB curve fitting toolbox for this question)

$$y = k_1 + \frac{k_2}{x}$$

(3 points)

- b) Plot the estimated function for $x \in [1, 10]$ on the same scatter plot of training data together with the validation data. Calculate SSE and R^2 for the training data. Calculate SSE for the validation data. **Include the plot in your report.**

(2 points)

- c) Estimate a polynomial model for the given data using MATLAB polynomial estimation functions, “polyfit” and “polyval”. Plot the SSE for validation data vs. polynomial order. **Include the plot in your report.** What polynomial order do you choose for the given data? Why?

(4 points)

5. (21 points)

The goal of this question is to identify dynamic systems using input-output data. Copy the files “runExam.m”, and “modeldata.mat” in your MATLAB current folder. Run the “runExam.m” script.

The following text should appear in your command window.

```
>> runExam
```

Please Enter the numeric part of your student number!:

Now, you should type your student number and press Enter. If your student number ends with an alphabetical letter, you should just type the numerical part of your student number, e.g. if your student number is 12345W, you should type 12345.

Then your data will be stored to MATLAB workspace. Data has been collected from three different dynamic systems. Input-output datasets are (u_1, y_1) , (u_2, y_2) , and (u_3, y_3) . The sampling frequency for all of the datasets has been 1 Hz. Identify polynomial models of the mentioned systems using MATLAB System Identification Toolbox. You should explicitly select a model as your final answer for each input-output dataset. You are supposed to answer the following question for each system.

- What is your selected model structure?
- Present the resulted plots and information related to validation procedure **only for your selected model structure**. (Hint: For instance residual analysis plots, poles and zeros plot, variance analysis information, etc.)
- What are the alternative system(s) for the data if you think there are any?

Save the final identification session for each dynamic system as “sys01.sid” for (u_1, y_1) , “sys02.sid” for (u_2, y_2) , and “sys03.sid” for (u_3, y_3) .

The points will be given based on the resulted model structures and their orders, as well as the justification of the identification path.

(7 points for identification of each system, 21 points in total)