ELEC-E8101 Digital and Optimal Control
Full exam 14. 12. 2016

- *Write the name of the course, your name, your study program, and student number to each answer sheet.*
- *There are five (5) problems and each one must be answered.*
- *No other literature except the Table of Formulas (Digital Control) is allowed. A function calculator can be used.*
- *The table of formulas must be returned, if you have received it from the exam supervisor.*
- *Mark clearly FULL EXAM on the answer sheet.*

Max 6 points / problem

1. Let us consider the system $\left(q^2 + 0.4q\right) y(k) = u(k)$.

a. For which values of K in the proportional controller $u(k) = K\left(u_c(k) - y(k)\right)$ is the closed-loop system stable? The term u_c denotes the reference signal. (3 p)

b. Determine the stationary error $u_c - y$ when u_c is a unit step and $K = 0.5$. (3 p)

2. Consider the discrete-time system

$$x(k+1) = \begin{bmatrix} 0.5 & -0.5 \\ 0 & 0.25 \end{bmatrix} x(k) + \begin{bmatrix} 6 \\ 4 \end{bmatrix} u(k)$$

$$y(k) = \begin{bmatrix} 2 & -4 \end{bmatrix} x(k)$$

a. Is the system observable? What does the answer mean? (2 p)
b. Is the system reachable? What does the answer mean? (2 p)
c. Construct a state-feedback control law to the system, which has the *dead-beat* tuning. Is the controlled system stable? (2 p)

3. Consider the scalar system

$$x(k+1) = ax(k) + bu(k), \quad x(0) = x_0 \quad (a\text{ , }b\text{ and }x_0\text{ are real numbers})$$

with the cost criterion to be minimized

$$J_0 = \frac{1}{2}x_N^2 + \frac{1}{2}\sum_{k=0}^{N-1}\left(x(k)^2 + 3u(k)^2\right)$$

The final state is free.

TURN PAPER!

Let N = 2. Write the equations, from which the optimal values for *u(k)* can be calculated. You do not have to solve the equations, but you have to write the solution in a clear form or algorithm, such that the values for *u* could easily be determined by a calculator or computer.

(6 p)

4. Consider the process

$$y(k+1)+ay(k)=bu(k)+e(k+1)+ce(k)$$

where a, b and c are constants, $|c|<1$ and e is zero mean white noise with variance 1.

a. Design a minimum variance control law to the system. What is the variance of the closed loop output signal y? (3 p)

b. It would seem wise to replace the noise terms in the system equation by $e_1(k+1)=e(k+1)+ce(k)$ and to design a minimum variance control law for this. Do so. Comments? (3 p)

5.

You are working in a company called *Full-service automation house*, which does automation projects for the industry. Your client asks for a short written explanation to the following concepts, which appear in your documents. Write those short descriptions. Please note that the client is somewhat aware of continuous control theory and automation, but he does not know digital control. Also, he does not want to read long explanations.

- pulse transfer function	(1 p)
- coloured noise	(1 p)
-hidden oscillations	(1 p)
- zero order hold	(1 p)
-ARMAX-model	(1 p)
- windup and antiwindup	(1 p)

Hint: These formulas can be used, if needed.

$$J_i=\frac{1}{2}x_N^T S_N x_N+\frac{1}{2}\sum_{k=i}^{N-1}\left(x_k^T Q_k x_k+u_k^T R_k u_k\right)$$

$$S_k=A_k^T\left[S_{k+1}-S_{k+1}B_k\left(B_k^T S_{k+1}B_k+R_k\right)^{-1}B_k^T S_{k+1}\right]A_k+Q_k,\ k<N,\ S_N \text{ given}$$

$$K_k=\left(B_k^T S_{k+1}B_k+R_k\right)^{-1}B_k^T S_{k+1}A_k,\quad k<N$$

$$J_i^*=\frac{1}{2}x_i^T S_i x_i$$

$$u_k=-K_k x_k,\quad k<N$$