1:	2:	3:	Extra:	**Total 1-3:**	**/ 23**
4:	5:	6:		**Total 4-6:**	**/ 20**

Aalto ME-C3100 Computer Graphics, Fall 2015

Lehtinen / Kemppinen, Ollikainen, Puomio

Tentti/Exam — Midterm 2/Välikoe 2, December 16 2015

Name, student ID: ______________________________

Allowed: Two two-sided A4 note sheets, calculators (also symbolic). Turn your notes in with your answers. Write your answers in either Finnish or English.

The following applies to students who are taking the class in fall 2015. If you have not taken the midterm (välikoe) in October 2015, you must answer *all* questions. Questions 1-3 cover the material from the 2nd half of class after October's midterm. Questions 4-6 cover the material from the first midterm. Only answer 4-6 if 1) you didn't take the midterm, or 2) wish to raise your midterm score. You're guaranteed not to make yourself worse off if you try: we'll scale the points accordingly and count only the better result.

1 Rendering Basics [/ 6]

1.1 Ray Casting vs. Rasterization [/ 4]

Give pseudocode for rendering an image using a ray caster and a rasterizer.

Ray Casting

Rasterization

1.2 Ray Tracer vs. Rasterizer Working Sets [/ 2]

What are the main differences between the working sets in rasterization and ray casting? For both algorithms:
a) what data needs to be kept in memory for random access during rendering? [**/ 1**]

b) what data can be processed in a stream (= computed once and then forgotten)? [**/ 1**]

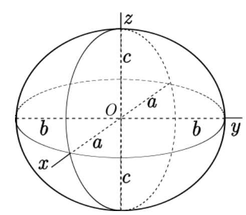
2 Ray Casting/Tracing and Rasterization [/ 11]

A ray with origin $\mathbf{o} = (o_x, o_y, o_z)$ and direction $\mathbf{d} = (d_x, d_y, d_z)$ is represented by $\mathbf{p} = \mathbf{o} + t\mathbf{d}$ in world coordinates.

2.1 Transforming Rays [/ 2]

A rigid object has a 4×4 object-to-world affine transformation matrix $\mathbf{M}$. Transform the world space ray $(\mathbf{o}, \mathbf{d})$ from to the object space ray $(\mathbf{o}', \mathbf{d}')$. Give the formulae for $\mathbf{o}'$, $\mathbf{d}'$. Pay attention to what you do with the homogeneous coordinates! Do not normalize the resulting vectors.

2.2 Ray-Ellipsoid Intersection [/ 5]



An axis-aligned ellipsoid centered at the origin is defined by $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$, where $a, b, c > 0$ are constants.

a) What is special about the ellipsoid when $a = b = c$? [/ **1**]

b) Derive the formula for intersecting a ray with the ellipsoid. The result is a quadratic equation in t with coefficients A, B, C. Give the expressions for A, B, C. *It is sufficient (and much easier!) to write the solution in terms of matrices and vectors — in the form $\mathbf{p}(t)^T\mathbf{M}\mathbf{p}(t) = 1$, where $\mathbf{p}(t)$ is the vector expression for the ray in terms of its origin and direction and $\mathbf{M}$ is a particular matrix — instead of expanding all the coefficients out. You do not need to solve for the actual intersections given A, B, C, we assume the quadratic solution formula is known.* [/ **4**]

2.3 Rasterization: Edge Functions [/ 1]

Edge functions $e_i(x, y) = a_i x + b_i y + c_i$ are 2D line equations that are computed from the three edges ($i = 1, 2, 3$) of a projected triangle. What is the mathematical condition that holds when a pixel/sample at (x, y) is inside the triangle?

2.4 Edge Functions [/ 3]

Given two projected vertices (x_1, y_1) and (x_2, y_2), derive the coefficients a, b, c for an edge function $e(x, y) = ax + bx + c$ that separates the plane into two half-spaces such that $e(x, y) = 0$ when the point (x, y) lies on the line defined by the two vertices, and has a positive value on one side (and resp. a negative value on the other side). It doesn't matter which side you choose to be positive. Note that the result is *not* unique: any positive non-zero multiple of a, b, c will give the same classification. *Hint: A 90 degree plane rotation is pretty simple.*

3 Shading and Sampling [/ 6]

3.1 The BRDF [/ 1]

The BRDF stands for "Bidirectional Reflectance Distribution Function". It is often denoted by $f_r(\mathbf{l}, \mathbf{v})$, where $\mathbf{l}$ is incident (light) direction and $\mathbf{v}$ is the outgoing (viewing) direction. What does the value $f_r(\mathbf{l}, \mathbf{v})$ tell you, in simple intuitive terms?

3.2 Diffuse Reflectance [/ **1**]

How does the BRDF of an ideally diffuse surface vary with **l** and **v**? (Pay attention here!)

3.3 Aliasing [/ **4**]

a) What is meant by pre-aliasing and post-aliasing? [/ **2**]

b) How does multisampling differ from supersampling? Why is it useful? [/ **2**]

4 Linear Algebra [/ 7]

4.1 Affine Transforms [/ 3]

What happens to lines under a linear transformation? What about a pair of parallel lines? How is the effect of an affine transform different, if at all?

4.2 Translation is not linear [/ 4]

Give the definition for a linear operation. Then, using your definition, show that translation $T_p(x) \mapsto x + p$ is *not* a linear operation in $\mathbb{R}^n$.

5 Numerical Integration of ODEs [/ 5]

a) What do you hope to accomplish when you use stiff springs for modeling cloth? [/ **1**]

b) The explicit Euler method has problems with stiff springs. What is the typical manifestation of this? [/ **1**]

c) Why does it make sense to write higher-order ODEs as first-order systems? Give one reason. [/ **1**]

d) What does the term *order* mean when talking about a numerical ODE solver? In big-Oh notation, how does the error E of an nth order solver behave with the step size h if the length of the simulated time interval is kept fixed? Also write out in words what this means for a 2nd order method. [/ **2**]

6 Splines [/ 8]

6.1 Converting between spline types [/ 2]

A cubic Bézier curve is given by

$$\mathbf{P}(t) = \mathbf{G}_B\,\mathbf{B}_B\,\mathbf{T}(t) = \begin{pmatrix}\mathbf{P}_1 & \mathbf{P}_2 & \mathbf{P}_3 & \mathbf{P}_4\end{pmatrix} \begin{pmatrix}1 & -3 & 3 & -1\\ 0 & 3 & -6 & 3\\ 0 & 0 & 3 & -3\\ 0 & 0 & 0 & 1\end{pmatrix} \begin{pmatrix}1\\ t\\ t^2\\ t^3\end{pmatrix},$$

where the $\mathbf{P}_i$ are the control points given as column vectors. A Catmull-Rom spline is given by a similar expression, with the matrix $\mathbf{B}_B$ replaced by

$$\mathbf{B}_{CR} = \frac{1}{2}\begin{pmatrix}0 & -1 & 2 & -1\\ 2 & 0 & -5 & 3\\ 0 & 1 & 4 & -3\\ 0 & 0 & -1 & 1\end{pmatrix}.$$

Write down the matrix expression for converting the Bézier spline to a Catmull-Rom spline. I.e., given the control point matrix $\mathbf{G}_B$ for a Bézier curve, what control point matrix $\mathbf{G}_{CR}$ do you need to feed into the Catmull-Rom spline so that you get the same curve? Write the equation down symbolically (no need to compute actual values for the matrices). *(See the extra credit section for a chance for more points.)*

6.2 Derivatives of Catmull-Rom Curves [/ 6]

Using the definition from above, write down a similar matrix-vector expression for the derivative $\mathbf{P}'(t)$ of the Catmull-Rom spline, i.e., give a matrix $\mathbf{B}'_{CR}$ such that evaluating $(\mathbf{P}_1\ \mathbf{P}_2\ \mathbf{P}_3\ \mathbf{P}_4)\ \mathbf{B}'_{CR}\ \mathbf{T}(t)$ gives the tangent $\mathbf{P}'(t)$ of the Catmull-Rom spline. **Hint:** You can do this any way you like. One way is to expand out the C-R basis polynomials $\mathbf{B}_{CR}\mathbf{T}(t)$, differentiate them, collect the terms, and regroup them into a new matrix. *(Continue onto the next blank page if necessary.)*

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A Extra Credit

No partial credit for extra credit questions. Extra credit is counted towards the score of questions 1-3 only.

A.1 Importance Sampling for Phong Lobes [/ 10]

A canonical Phong lobe (one that points towards the North pole) with exponent q is given in spherical coordinates θ (polar angle) and ϕ (azimuth) by $P(\theta, \phi; q) = \cos^q \theta$. Given a uniformly distributed random point in the 2D unit square, i.e., two uniformly distributed random numbers $x_1 \in [0, 1]$ and $x_2 \in [0, 1]$, derive the formulae for a mapping of the square to the hemisphere such that the resulting density of points on the hemisphere is proportional to $P(\cdot, \cdot; q)$. Explicitly: give the formulae for the polar coordinates $\theta(x_1, x_2)$ and $\phi(x_1, x_2)$ such that the preceding holds. Remember the usual relationship $\mathrm{d}A = \sin\theta \, \mathrm{d}\theta \, \mathrm{d}\phi$ between the spherical area measure $\mathrm{d}A$ and the polar coordinate measure.

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