1:	2:	3:	Extra:	**Total 1-3:**	**/ 24**
4:	5:	6:		**Total 4-6:**	**/ 17**

Aalto ME-C3100 Computer Graphics, Fall 2014

Lehtinen / Kemppinen, Seppälä

Tentti/Exam — Midterm 2/Välikoe 2, December 17 2014

Allowed: Two two-sided A4 note sheets, calculators (also symbolic). Turn your notes in with your answers. Write your answers in either Finnish or English.

The following applies to students who are taking the class in fall 2014. If you have not taken the midterm (välikoe) in October 2014, you must answer all questions. Questions 1-3 cover the material from the 2nd half of class after October's midterm. Questions 4-6 cover the material from the first midterm. Only answer 4-6 if 1) you didn't take the midterm, or 2) wish to raise your midterm score. You're guaranteed not to make yourself worse off if you try (we'll scale the points accordingly).

Name, student ID: ____

1 Rendering Basics [/ 6]

1.1 Ray Casting vs. Rasterization [/ 4]

Give pseudocode for rendering an image using a ray caster and a rasterizer.

Ray Casting	Rasterization

1.2 Working Set [/ 2]

What are the main differences between the working sets in rasterization and ray casting? I.e., for both cases: what data needs to be kept in memory for random access during the entire rendering, and what data can be processed in a stream?

2 Ray Casting/Tracing and Rasterization [/ 12]

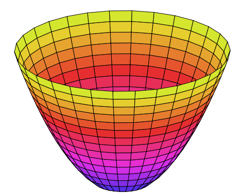
A ray with origin $\mathbf{o} = (o_x, o_y, o_z)$ and direction $\mathbf{d} = (d_x, d_y, d_z)$ is represented by $\mathbf{p} = \mathbf{o} + t\mathbf{d}$.

2.1 Ray-Plane Intersection [/ 2]

An infinite plane may be represented implicitly as the set of 3D points $\mathbf{p} = (x, y, z)$ for which $\mathbf{p} \cdot \mathbf{n} + d = 0$. Here $\mathbf{n}$ is the plane normal, and d is a real constant. Derive the formula for intersecting a ray and a plane. You can assume that the ray direction is not tangent to the plane. [/ **2**]

2.2 Ray-Paraboloid Intersection [/ 4]

An axis-aligned *elliptic paraboloid* (see picture) is defined by the equation $\frac{x^2}{a^2} + \frac{y^2}{b^2} = \frac{z}{c}$, where a, b, c are constants. Derive the formula for intersecting a ray with the paraboloid. The result is a quadratic equation $At^2 + Bt + C = 0$ with coefficients A, B, C. Give the expressions for A, B, C. *You do not need to solve for the actual intersections given the coefficients, we assume the quadratic solution formula is known.*



2.3 Rasterization: Edge Functions [/ 1]

Edge functions $e_i(x, y) = a_i x + b_i y + c_i$ are 2D line equations that are computed from the three edges ($i = 1, 2, 3$) of a projected triangle. What is the mathematical condition that holds when a pixel/sample at (x, y) is inside the triangle?

2.4 Rasterization Using Edge Functions [/ 5]

Give pseudocode for rasterizing a triangle using edge functions, starting before projection. Your code should use screen bounding boxes for avoiding testing all pixels on the screen and include z-buffering for visibility. You can assume the triangle has a constant color and that clipping has been performed already.

3 Shading and Sampling [/ 6]

3.1 Irradiance [/ 2]

How does the irradiance incident on a surface change as a function of the angle between the surface normal $\mathbf{n}$ and incident light direction $\mathbf{l}$? Which special case does one need to consider?

3.2 The BRDF [/ 1]

The BRDF stands for "Bidirectional Reflectance Distribution Function". It is often denoted by $f_r(\mathbf{l}, \mathbf{v})$, where $\mathbf{l}$ is incident (light) direction and $\mathbf{v}$ is the outgoing (viewing) direction. What does the value $f_r(\mathbf{l}, \mathbf{v})$ tell you?

3.3 Aliasing [/ 3]

a) What is meant by pre-aliasing and post-aliasing? [/ **2**]

b) Name one method for avoiding pre-aliasing. [/ **1**]

4 Linear Algebra [/ 6]

4.1 Linearity [/ 2]

What properties characterize a linear function (operator) $L(x)$, with $x \in \mathbb{R}^n$? Write down one or two equations.

4.2 Affine Transforms [/ 2]

How does a linear transform differ from an affine transform? Specifically, compare what happens to the origin $\mathbf{0} \in \mathbb{R}^n$ under a linear transformation as opposed to an affine transformation.

4.3 Rotation Matrices [/ 2]

Rotation matrices are characterized by $\mathbf{R}^{\mathrm{T}}\mathbf{R} = \mathbf{I}$ and $\det(\mathbf{R}) = 1$. Show that the matrix $\mathbf{RS}$ is a rotation matrix whenever $\mathbf{R}$ and $\mathbf{S}$ are rotation matrices. (Remember the elementary properties of determinants.)

5 Numerical Integration of ODEs [/ 5]

a) What do you hope to accomplish when you use stiff springs for modeling cloth? [/ **1**]

b) The explicit Euler method has problems with stiff springs. What is the typical manifestation of this? [/ **1**]

c) Why does it make sense to write higher-order ODEs as first-order systems? Give one reason. [/ **1**]

d) What does the term *order* mean when talking about a numerical ODE solver? In big-Oh notation, how does the error E of an nth order solver behave with the step size h if the length of the simulated time interval is kept fixed? Also write out in words what this means for a 2nd order method. [/ **2**]

6 B-Splines [/ 6]

A cubic B-spline is a piecewise cubic curve defined by a "sliding window" of 4 control points. Each set of 4 contiguous control points defines one segment of the spline. Consider the 2-segment spline given by 5 control points $\{P_1, \ldots, P_5\}$. The ith segment ($i = 1, 2$) is given by $P_i(t) = \sum_{j=1}^{4} B_j(t)\, P_{i+j-1}$, where the basis functions are defined as $B_1(t) = \frac{1}{6}(1-t)^3$, $B_2(t) = \frac{1}{6}(3t^3 - 6t^2 + 4)$, $B_3(t) = \frac{1}{6}(-3t^3 + 3t^2 + 3t + 1)$, $B_4(t) = \frac{1}{6}t^3$. Show that cubic B-splines are C^2 continuous, i.e., that $P_1(1) = P_2(0)$, and that also the tangents and second derivatives of the segments P_1 and P_2 agree at the join.

A Extra Credit

No partial credit for extra credit questions. Extra credit is counted towards the score of questions 1-3 only.

A.1 Importance Sampling for Phong Lobes [/ 10]

A canonical Phong lobe (one that points towards the North pole) with exponent q is given in spherical coordinates θ (polar angle) and ϕ (azimuth) by $P(\theta, \phi; q) = \cos^q \theta$. Given a uniformly distributed random point in the 2D unit square, i.e., two uniformly distributed random numbers $x_1 \in [0, 1]$ and $x_2 \in [0, 1]$, derive the formulae for a mapping of the square to the hemisphere such that the resulting density of points on the hemisphere is proportional to $P(\cdot, \cdot; q)$. Explicitly: give the formulae for the polar coordinates $\theta(x_1, x_2)$ and $\phi(x_1, x_2)$ such that the preceding holds. Remember the usual relationship $\mathrm{d}A = \sin\theta \,\mathrm{d}\theta \,\mathrm{d}\phi$ between the spherical area measure $\mathrm{d}A$ and the polar coordinate measure.

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