MEC-E1020 Fluid dynamics - Exam - 28.10.2016

Choose one option from each topic group, so that you have at maximum 2×option 1 and 2×option 2. In order to pass, you need to get at least 10 points in total and at least 4 points for options 1 and 4 points for options 2. **Write each option on a separate piece of paper.**

Fundamental equations and their solutions

Option 1

1. Which are the three fundamental principles used to derive the governing equations? How would you describe these principles verbally? (1p)

2. An equation is given as

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \nu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}\right) + g_x \ .$$

   What is this equation and under which conditions (type of flow/fluid) is this equation valid? What is the physical meaning of each term in the equation? Be specific in both answers. (2p)

3. The total rate of work per volume done by the stress τ_{ij} can be divided into two contributions:

$$\frac{\partial}{\partial x_j}\left(u_i\tau_{ij}\right) = u_i\frac{\partial \tau_{ij}}{\partial x_j} + \tau_{ij}\frac{\partial u_i}{\partial x_j} \ .$$

   What is the physical meaning of the contributions on the right hand side and how do they affect the energy of the flow? How can you justify this? (3p)

Option 2

A product has a small gap (height b) between two parallel plates, in which the fluid is driven by a specific pressure gradient dp/dx. Your colleague claims that the flow rate doubles for the same pressure gradient, if we double the spacing between the plates.

1. By starting from the continuity condition and the momentum equations for incompressible flow convince your colleague to reconsider the claim and give the correct factor of change in the flow rate. Clearly state the assumptions that you make in simplifying the equations. (4p)

2. Your colleague gets really confused and wants to know the physical reason, why the flow rate does not simply double. Write the force balance for a fluid element with a length Δx and spanning the whole gap and try to explain the change in the flow rate based on the behaviour of the shear stress on the wall as the gap increases. (2p)

Boundary layers and related flows

Option 1

1. Explain, why the boundary layer developing on the surface of a body is displacing the streamlines away from the body. How can you evaluate the size of this displacement, if you know the velocity profile in the boundary layer? (2p)

2. The nondimensional shear stress on a flat plate is given with a formula

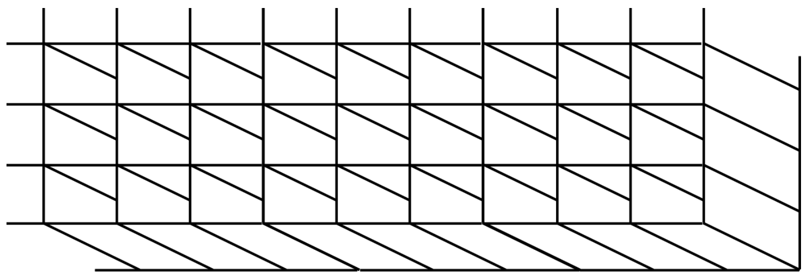
$$c_\mathrm{f} = \frac{0.664}{\sqrt{\mathrm{Re}_x}} \ .$$

   Explain the origin of this formula including the assumptions behind the formula and the limitations related to its use. (2p)

3. Explain, how the pressure gradient affects the shape of the velocity profile in a boundary layer. How can this effect be explained based on the momentum equation on the wall? (2p)

Option 2

Your task is to analyse the pressure drop caused by a honeycomb flow straightener to be used in a low-speed wind tunnel that has a rectangular cross-section. The straightener consists of a large number of cells, which have also a rectancular cross-section of 1 cm by 1 cm, a length of 10 cm and a very thin plate thickness (negligible pressure drag).



1. Estimate the pressure drop caused by the straightener, when the flow speed is 10 m/s. The pressure drop is equal to the total drag of the flow straightener divided by the cross-section of the tunnel. The air in the tunnel has a density of 1.2 kg/m^3 and a dynamic viscosity of 1.8×10^{-5} kg/(m s). (3p)

2. Draw a sketch of the axial velocity profile right after the straightener normalised by the incoming velocity. (1p)

3. In order to improve the performance in terms of straightening the flow, technicians propose two solutions: a) reduce the cell cross-section to 0.5 cm by 0.5 cm or b) increase the length to 20 cm. Which option would you favor, if these are considered only from the point of view of the pressure drop (as low as possible) and uniformity of the axial flow behind the flow straightener (as uniform as possible)? Convince the technicians with proper analysis. (2p)

Instability and turbulence

Option 1

1. Explain, what the eddy viscosity hypothesis means. (1p)
2. Explain the process of deriving the RANS-equations, how the equations for the mean flow compare to the Navier-Stokes equations and what is the role of the turbulence models. (3p)
3. Explain, what we mean by the energy cascade and what is the role of the viscosity in this? (2p)

Option 2

Let's assume that the characteristics of a turbulent boundary layer are given by

$$\delta \approx \frac{0.16x}{\mathrm{Re}_x^{1/7}} \qquad c_f \approx \frac{0.027}{\mathrm{Re}_x^{1/7}} \ ,$$

where δ is the boundary layer thickness and c_f is the nondimensional shear stress on the wall. Study the structure of the boundary layer velocity profile at $x = 10\,\mathrm{m}$, when the flow speed is $10\,\mathrm{m/s}$, the density of the fluid is $1000\,\mathrm{kg/m^3}$ and the kinematic viscosity is $1.0 \times 10^{-6}\,\mathrm{m^2/s}$.

1. What is the boundary layer thickness δ at this location? (1p)
2. What is the boundary layer thickness δ^+ at this location? (2p)
3. Roughness does not have an adverse effect on the boundary layer, if the roughness elements on the wall remain inside the viscous sublayer. What is the maximum allowed height of the roughness as micrometers at this location, so that it does not have an adverse effect? How does the maximum allowed height change, if we move upstream or downstream on the wall? Justify the answer. What is the allowed height for different locations as a function of only x and the local Reynolds number Re_x? (3p)

Numerical techniques

Option 1

1. Explain what is the difference between verification and validation. (1p)

2. Describe the different sources of error in numerical predictions and the origins of the errors. (2p)

3. Explain in detail the process of quantifying the numerical uncertainty based on Richardson extrapolation. In which cases does this process fail? What risk is involved in the use of a minimum number of resolutions? (3p)

Option 2

The convection-diffusion equation

$$\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} = D\frac{\partial^2 T}{\partial x^2} \ ,$$

where u is known and T is unknown, should be discretised with backward difference in time and with centered differences for the spatial derivatives.

1. Write the difference approximations for all the derivatives in the equation. (2p)

2. Derive the orders of accuracy for these approximations. (3p)

3. Does this set of discretisations produce an explicit or implicit scheme. Justify your answer. (1p)