MEC-E1020 Fluid dynamics - Exam - 16.12.2016

Choose one option from each topic group, so that you have at maximum 2×option 1 and 2×option 2. In order to pass, you need to get at least 8 points in total and at least 4 points for options 1 and 2 points for options 2. If you are retaking only option 1 or option 2, you are free to choose the two tasks from any of the four topic groups.

Fundamental equations and their solutions

Option 1

1. The conservation of momentum is given by

$$\int_V \rho \frac{Du_i}{Dt} dV = \int_V \rho g_i dV + \int_A \tau_{ij} dA_j \ .$$

   Explain the process of deriving (no need to derive) the differential form

$$\rho \frac{Du_i}{Dt} = \rho g_i + \frac{\partial \tau_{ij}}{\partial x_j}$$

   starting from the integral form above. Specify each step clearly. What is the difference in the physical meaning between the differential and the integral forms? (3p)

2. An equation is given as

$$\rho \frac{D}{Dt} \left(\frac{1}{2} u_i^2 \right) = \rho \mathbf{g} \cdot \mathbf{u} + \frac{\partial}{\partial x_j} (u_i \tau_{ij}) + p (\nabla \cdot \mathbf{u}) - \phi \ .$$

   What is this equation and what is the physical meaning of each term in the equation? Be specific. (3p)

Option 2

A production line has a small gap (height b) between two parallel plates, in which the fluid is driven by the movement of the upper plate (at $y = b$) and pressure remains constant throughout the fluid. This could be a large belt running close to a surface.

1. Derive the velocity profile in the gap by starting from the continuity condition and the momentum equations for incompressible flow and determine the volume flow rate in the gap. Clearly state the assumptions that you make in simplifying the equations. (4p)

2. How does the shear stress vary in the gap? (1p)

3. What can you say about the balance of the fluid forces in this case? (1p)

Boundary layers and related flows

Option 1

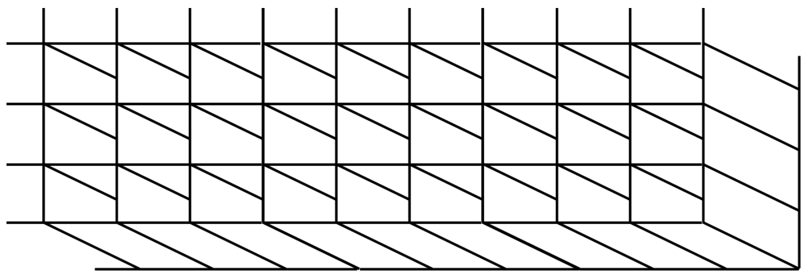
1. Explain, how the flow equations can be simplified in case of boundary layer flow. What are the relevant assumptions on the character of the flow that allows us to simplify the equations? (2p)

2. How can we exploit the parabolic character of the boundary layer equations, when solving the equations? How is the flow outside of the boundary layer taken into account? (2p)

3. Explain the process leading to the separation of the boundary layer. (2p)

Option 2

Your task is to analyse a honeycomb flow straightener to be used in a wind tunnel to reduce cross-stream velocity components. The tunnel has a rectangular cross-section. The straightener consists of a large number of cells, which have also a rectancular cross-section of 1 cm by 1 cm, a length of 10 cm and a very thin plate thickness. Assume that you only need to consider the axial velocity component through the honeycomb and that the on-coming axial flow is uniform.



1. Sketch the evolution of the axial velocity profile within a cell. (1p)

2. Estimate the limit for the flow velocity, if the threshold is that the contraction of the effective flow area at the trailing edge of the flow straightener should not be more than 20 percent of the actual cross-sectional area. Is this a lower or upper limit (justify your answer)? The air in the tunnel has a density of $1.2\,\mathrm{kg/m^3}$ and a dynamic viscosity of $1.8 \times 10^{-5}\,\mathrm{kg/(m\,s)}$. (3p)

3. In order to improve the performance in terms of straightening the flow, technicians propose two solutions: a) reduce the cell cross-section to 0.5 cm by 0.5 cm or b) increase the length to 20 cm. Analyse the options in terms of the uniformity of the axial flow behind the flow straightener (as uniform as possible) at a flow speed of 20 m/s. How do the results compare with the original setting? (2p)

$$\delta_{99} = \frac{4.9x}{\sqrt{\mathrm{Re}_x}} \qquad \delta^* = \frac{1.72x}{\sqrt{\mathrm{Re}_x}}$$

Instability and turbulence

Option 1

1. Explain, what we mean by Reynolds stress, where does it originate from and how is the Reynolds stress typically modelled. (3p)

2. Describe in detail the structure of a turbulent boundary layer. How does the pressure gradient outside of the boundary layer influence the velocity distribution inside a turbulent boundary layer? (3p)

Option 2

Let's assume that the characteristics of a turbulent flat plate boundary layer are given by

$$\delta \approx \frac{0.16x}{\mathrm{Re}_x^{1/7}} \qquad c_f \approx \frac{0.027}{\mathrm{Re}_x^{1/7}} \ ,$$

where δ is the boundary layer thickness and c_f is the nondimensional shear stress on the wall. Study the boundary layer velocity profile at $x = 10\,\mathrm{m}$, when the flow speed is $10\,\mathrm{m/s}$, the density of the fluid is $1000\,\mathrm{kg/m^3}$ and the kinematic viscosity is $1.0 \times 10^{-6}\,\mathrm{m^2/s}$.

1. Evaluate in which parts of the boundary layer the points with a distance of $13\,\mu\mathrm{m}$, $320\,\mu\mathrm{m}$ and $1\,\mathrm{cm}$ from the wall are located. (3p)

2. Evaluate the dimensional velocity u at the three points given above. (3p)

Numerical techniques

Option 1

1. What do we mean by an explicit or an implicit scheme? (2p)
2. What do we mean by truncation error? How can we estimate this based on simulation results? (2p)
3. Give an example of a modelling error. How can we estimate the size of the modelling error? (2p)

Option 2

The convection-diffusion equation

$$\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} = D\frac{\partial^2 T}{\partial x^2} \ ,$$

where u is known and T is unknown, should be discretised with forward difference in time and with backward difference for the convective term and centered difference for the diffusion term.

1. Write the difference approximations for all the derivatives in the equation. (2p)
2. Derive the orders of accuracy for these approximations. (3p)
3. Does this set of discretisations produce an explicit or implicit scheme. Justify your answer. (1p)