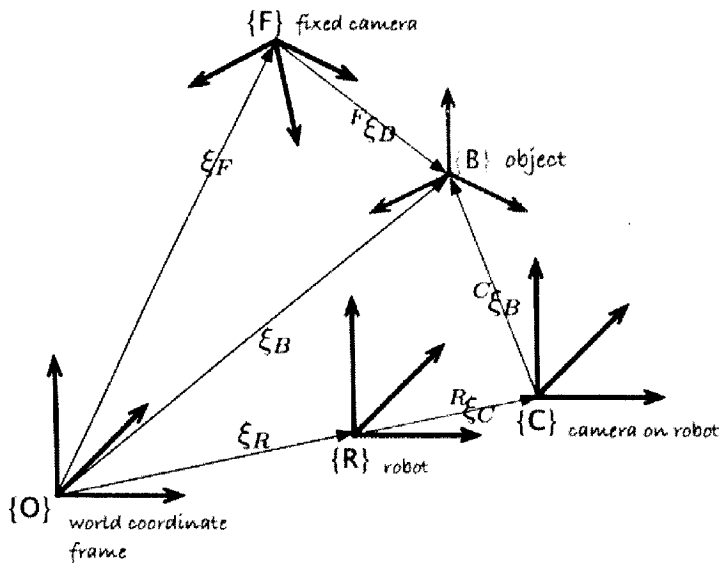


ELEC-C1320 – Robotiikka, Exam 13.12.2016 (3 hours)

It is allowed to use a calculator and a book of mathematical equations (e.g. MAOL) in the exam.

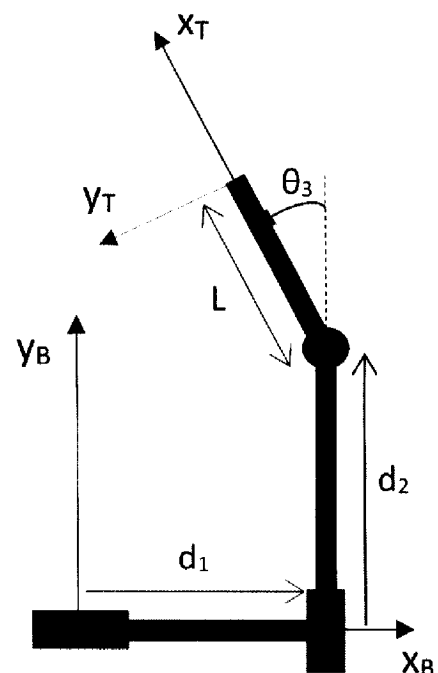
1. In the figure below, known relative transformations between some of the key coordinate frames in a robot cell are described with the ξ -symbols. Based on the figure, give all the (unique) kinematic paths to describe the position and orientation of the object {B} with respect to the fixed camera frame {F}. (9 points)



2. In the figure a planar PPR robot is shown. The orientation of the last link when the joint variable θ_3 is zero is indicated with the dashed line. The direction of positive rotation of θ_3 is indicated with the arrow in the figure.

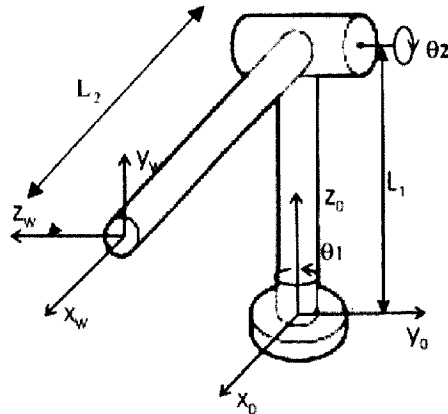
Solve the forward kinematics problem of the manipulator to describe the tool frame (T) with respect to the robot base frame (B). In other words, assign the link frames and provide the corresponding DenavitHartenberg-parameters in a table. It is your choice to use either the Standard or Modified DH-parameter convention. (18 points)

Hint: You can assign the link frames into the given figure where all the joint variables have a non-zero value.

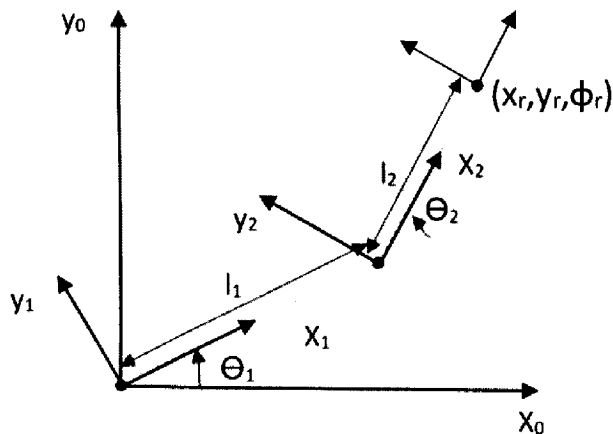


3. In the figure below a two degrees-of-freedom manipulator in its home/zero position is shown (upper arm oriented horizontally above the X_0 -axis). Both degrees-of-freedom (dof) are rotational (the first rotating the upper link on the horizontal plane, Θ_1 , and the second tilting the upper link with respect to the horizontal plane, Θ_2). Positive directions of rotations are also shown in the figure.

Find the inverse kinematic transform for the manipulator. (15 points)



4. Form the manipulator Jacobian matrix for the planar RR-manipulator shown in the figure below. (11 points)



The Jacobian matrix should obey the equation:

$$\begin{bmatrix} \dot{x}_r \\ \dot{y}_r \\ \dot{\phi}_r \end{bmatrix} = J_r(q) \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \end{bmatrix}$$

The forward kinematics model of the manipulator is:

$$x_r = l_1 c_1 + l_2 c_{12}$$

$$y_r = l_1 s_1 + l_2 s_{12}$$

$$\phi_r = \theta_1 + \theta_2$$

where c_{12} means $\cos(\theta_1 + \theta_2)$ and so on.

5. A weight of 3kg is fixed to the tip of last link (at the location of the origin of the H-frame) of the three-link manipulator shown in the figure below. The task is to calculate torques and forces affecting joints 1, 2 and 3 (due to gravity) in two different configurations of the manipulator arm. To solve the problem here you must utilize the Jacobian matrix of the manipulator. The joint configurations to be considered are:

a) $\theta_1=90.0^\circ$, $\theta_2=30.0^\circ$, $d_3=0.5\text{m}$ (the total length of the upper link is described by d_3) (6 points)

b) $\theta_1=90.0^\circ$, $\theta_2=90.0^\circ$, $d_3=0.5\text{m}$ (the total length of the upper link is described by d_3) (6 points)

The links itself are assumed to be weightless.

The gravitational acceleration vector is pointing in the direction of negative Z_R -axis and its value is 9.81 m/s^2 .

The Jacobian matrix of the manipulator is:

$$\begin{bmatrix} v_x \\ v_y \\ v_z \end{bmatrix} = J \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \\ \dot{d}_3 \end{bmatrix}, \quad J = \begin{bmatrix} -s\theta_1 c\theta_2 d_3 & -c\theta_1 s\theta_2 d_3 & c\theta_1 c\theta_2 \\ c\theta_1 c\theta_2 d_3 & -s\theta_1 s\theta_2 d_3 & s\theta_1 c\theta_2 \\ 0 & c\theta_2 d_3 & s\theta_2 \end{bmatrix}$$

