

## ELEC-E8101 Digital and Optimal Control

Full exam 30. 1. 2017

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- Write the name of the course, your name, your study program, and student number to each answer sheet.
  - There are five (5) problems and each one must be answered.
  - No other literature except the Table of Formulas (Digital Control) is allowed. A function calculator can be used.
  - The table of formulas must be returned, if you have received it from the exam supervisor.
  - Mark clearly FULL EXAM on the answer sheet.
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Max 6 points / problem

1. A continuous time system is discretized by using the sampling interval  $h$  and assuming zero order hold. Let the continuous time system have a pole at point  $z_1 = a + jb$ , where  $j$  is the imaginary unit and  $a, b$  are real constants. Where is the corresponding pole located in the discretized system? Discuss different possibilities when  $a$  and  $b$  vary. Consider especially oscillations and stability. How is the concept *Nyquist frequency* related here? (6 p)
2. Explain briefly the meaning of the following concepts (1 p each)
  - a. BIBO stability,
  - b. Reachability,
  - c. Observability,
  - d. Alias-effect,
  - e. Causal controller
  - f. "Integrator windup"-phenomenon
3. Write a state-space representation for the following discrete-time system and compute a dead-beat controller for it.

$$y(k) + 2y(k-1) + y(k-2) = 0.5u(k-2)$$

( $y$  and  $u$  are the output and input variables of the system). (3 p) representation, (3 p) controller

4. Consider the ARMAX-process

$$\begin{aligned} y(k+3) - y(k+2) + 0.5y(k+1) &= \\ &= u(k+1) + 0.5u(k) + 0.5e(k+3) + 0.4e(k+2) + 0.125e(k+1) \end{aligned}$$

where  $e(k)$  is zero-mean white noise with unit variance. Determine the minimum-variance controller. (6 p)

5. Present the equation of a continuous PID-controller in time and Laplace domains. Compute the discretized version of the controller in time and z-domains. (You can freely choose the discretization method.) (2p) PID equation, (4 p) discretized version