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CS-E3200 – Discrete Models and Search (5 cr)
Exam April 5, 2017

Write down on each answer sheet:
- Your name, degree program, and student number.
- The text: "CS-E3200 Discrete Models and Search 5.4.2017".
- The total number of answer sheets you are submitting for grading.
- At most 5 question numbers from the set $\{1,2,3,4,5,6\}$ that you want to be graded.

Note 1: You can write down your answers in either Finnish, Swedish, or English. Calculators are allowed.
Note 2: **Only 5 questions (out of 6) will be graded** and you need to **specify which five questions** should be graded.

1. Propositional logic: *(total: 10 points)*
   (a) Convert formula $\varphi := (\neg(p \wedge q) \leftrightarrow (r \wedge s)) \wedge \neg(\neg r \vee \neg s)$ into CNF. *(4 points)*
   (b) Use truth tables to prove that formula $\psi := (\neg p \vee \neg q)$ is a consequence of the formula φ above. *(3 points)*
   (c) Use resolution to prove the part (b) again, i.e., use resolution to prove that $\{\varphi\} \models \psi$. *(3 points)*

2. Consider the 9×9 Sudoku puzzle (as in Figure 1) where a partially filled 9×9 table is given, and you are asked to fill each blank cell with a number in $1,2,\ldots,9$ such that the set of numbers in each row, each column and each of the nine 3×3 small squares is equal to the set $\{1,\ldots,9\}$.
   Now, design a constraint satisfaction problem which models a given Sudoku puzzle instance. Describe your variables *(2 points)*, the domain of each variable *(2 points)*, and the constraints over those variables *(3 points)*. Now, extend your CSP model of Sudoku to any $n^2 \times n^2$ instance of Sudoku puzzle *(3 points)*. *(total: 10 points)*

3. Use initial assignment $\tau = \{a \mapsto 0, b \mapsto 1, c \mapsto 1, d \mapsto 0, e \mapsto 1\}$ to simulate 5 steps of the GSAT algorithm on the following clauses *(2 points/step)*. Describe the reasons for changes from step i to $i+1$. *(total: 10 points)*

$c_1 : a \vee c$	$c_6 : \neg a \vee \neg e$	$c_{11} : \neg b \vee \neg c \vee e$
$c_2 : a \vee \neg c \vee d$	$c_7 : b \vee \neg c \vee d$	$c_{12} : \neg b \vee \neg c \vee \neg e$
$c_3 : a \vee \neg c \vee \neg d$	$c_8 : b \vee \neg c \vee \neg d$	$c_{13} : \neg b \vee \neg e$
$c_4 : a \vee \neg c \vee e$	$c_9 : \neg b \vee c$	$c_{14} : d \vee e$
$c_5 : a \vee d \vee \neg e$	$c_{10} : \neg b \vee \neg c \vee d$	

4. Assume that x and y are integers in the range $[-10, 10]$ and $b_1, b_2, \ldots, b_5$ are binary variables. Do the followings using integer linear programming: *(total: 10 points)*
   (i) Express $b_1 = 1$ if and only if $x \geq 3$. *(2 points)*
   (ii) Express $b_2 = 1$ if and only if $x \leq -3$. *(2 points)*
   (iii) Use b_3, b_4 and b_5 to express $b_3 = 1$ if and only if $y = 2$. *(2 points)*
   (iv) Use b_1, b_2 and b_3 as above to express "$|x| \geq 3$ if and only if $y = 2$". *(2 points)*
   (v) Obtain concrete values for big M's that you have used above. *(2 points)*

5. Consider the integer linear program below. Give its linear relaxation *(1 point)*. Transform the linear relaxation to standard form *(2 points)*. Write this relaxed problem as a Simplex tableau with a basis that corresponds to when $x_1 = x_2 = 0$ *(2 points)*. Given such a starting point, simulate Simplex algorithm to obtain the optimal solution to the relaxed problem *(5 points)*. *(total: 10 points)*

$$\begin{aligned} &\max\ 90x_1 + 75x_2 \text{ s.t.} \\ &3x_1 + 2x_2 \leq 66 \\ &9x_1 + 4x_2 \leq 180 \\ &2x_1 + 10x_2 \leq 200 \\ &x_1, x_2 \geq 0 \\ &x_1, x_2 \text{ are integers} \end{aligned}$$

6. Consider the *Hamiltonian Completion Problem* where you are given a graph $G = (V;E)$ and asked to find a set $E' \subseteq V \times V$ with minimum size such that the graph $G' = (V; E \cup E')$ is Hamiltonian (i.e., there exists a tour in G' which passes each vertex exactly once and returns to its initial location). Also, assume that you have a function $H : V \times E \times \mathbb{N} \mapsto \{\top, \bot\}$ which takes a graph $G = (V;E)$ plus a number k and returns $\top$ if and only if there exists a set E' with $|E'| \leq k$ such that $G' = (V; E \cup E')$ is Hamiltonian. Now, design an algorithm that uses function H to solve the Hamiltonian Completion Problem in polynomial time (time spent inside function H is not counted). That is, your algorithm should take a graph $G = (V;E)$ and print an E' of minimum size such that $G' = (V; E \cup E')$ is Hamiltonian. *(10 points)*

			4	6				3
			3			4	1	
	1	4						
8		1	7		2		3	
	2		8		3	6		9
						9	2	
	8	3			6			
5				4	9			

Figure 1. A sample instance of a 9×9 Sudoku puzzle. It consists of nine 3×3 squares that, together, form a 9×9 square. Some cells are pre-filled with numbers 1 to 9. A solution should fill blank cells with numbers 1 to 9 so that the set of numbers in each row, each column, and each small 3×3 square is equal to the set $\{1, \ldots, 9\}$. While you are free to spend your time on solving this Sudoku instance, you would not receive any points for doing so ;-)