ELEC-E8116 Model-based control systems
Intermediate exam 1. 16.2.2016

- *Write the name of the course, your name, your study program, and student number to each answer sheet.*
- *There are three (3) problems and each one must be answered.*
- *No literature is allowed. A function calculator can be used.*

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1. Explain briefly the following concepts

- Singular value decomposition and singular values
- Conservative control law
- Robust stability
- Small gain theorem
- Pole polynomial of a multivariable linear system

2. Consider a multivariable control configuration.

a. Draw a schema of the ”two-degrees-of-freedom” control configuration.
Define the concepts *loop transfer function, sensitivity function* and *complementary sensitivity function* for it.

b. Prove the identity $A(I+BA)^{-1} = (I+AB)^{-1}A$. What conditions (e.g. related to matrix dimensions) must be fulfilled in order the equality to be correct.

c. Let G be the (multivariable) process model and F_y the feedback controller, Prove that it generally holds

$$(I+GF_y)^{-1}GF_y = GF_y(I+GF_y)^{-1}$$

3. Consider the scalar system

$$\dot{x}(t) = 2x(t) + 2u(t) + w(t)$$
$$y(t) = 3x(t) + v(t)$$

in which the noise terms w and v are white noise with intensity 1 and 5, respectively.

a. Form the (stationary) Riccati equation and solve it.

b. Calculate the Kalman-gain and form the (stationary) Kalman-filter. Calculate the poles of the estimator.

c. How do the poles move, if the intensity of the measurement noise grows tenfold? Explain.

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Hint.

$$\dot{x} = Ax + Bu + Nv_1$$
$$y = Cx + Du + v_2$$

$$\dot{\hat{x}}(t) = A\hat{x}(t) + Bu(t) + K\left(y(t) - C\hat{x}(t) - Du(t)\right)$$
$$K = (PC^T + NR_{12})R_2^{-1}$$
$$AP + PA^T - \left(PC^T + NR_{12}\right)R_2^{-1}\left(PC^T + NR_{12}\right)^T + NR_1N^T = 0$$