ELEC-E8116 Model-based control systems
Intermediate exam 2. 5.4.2016

- *Write the name of the course, your name, your study program, and student number to each answer sheet.*
- *There are three (3) problems and each one must be answered.*
- *No literature is allowed. A function calculator can be used.*

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1. Consider a SISO-process with the transfer function

$$G(s) = \frac{s + \frac{1}{T_1}}{\left(s + \frac{1}{T_2}\right)\left(s + \frac{1}{T_3}\right)}, \quad T_1 < 0, \quad T_2 < 0, \quad T_3 > 0$$

Explain, what kind of fundamental limitations on control performance can be stated for this system? (Present also some calculations; the formulas in the end of the problems can be of help.)

2. a. Consider a MIMO system with the transfer function matrix

$$G(s) = \begin{bmatrix} \frac{2}{s+1} & \frac{3}{s+2} \\ \frac{1}{s+1} & \frac{1}{s+1} \end{bmatrix}$$

Calculate the *Relative Gain Array* RGA at zero frequency in the above example case. What conclusions can be made with respect to control?

b. Consider a SISO-system in a one-degree-of-freedom control configuration. The connection between the real (G_0) and nominal (G) system is

$$G_0(s) = G(s)(1 + \Delta_G(s))$$

Derive a condition for the system to be robustly stable.

3. Consider the 1.order process $G(s) = \frac{1}{s-a}$, which has a realization

$$\dot{x}(t) = ax(t) + u(t)$$
$$y(t) = x(t)$$

so that the state is directly measurable . It is desired to find a control that minimizes the criterion

$$J = \int_0^\infty \left(x^2 + Ru^2\right)dt \qquad (R > 0)$$

Determine the solution and determine the closed loop state equation. Is the closed loop system stable, when the process is i. stable, ii. unstable?

Some formulas that might be useful:

$$\int_0^\infty \log \left|S(i\omega)\right| d\omega = \pi \sum_{i=1}^{M} \mathrm{Re}\,(\mathrm{p_i})$$

$$\left|W_T(p_1)\right| \le 1 \quad \Rightarrow \quad \omega_0 \ge \frac{p_1}{1 - 1/T_0}$$

$$\left|W_S(z)\right| \le 1 \quad \Rightarrow \quad \omega_0 \le \left(1 - 1/S_0\right) z$$

$$\dot{x} = Ax + Bu, \quad t \ge t_0$$

$$J(t_0) = \frac{1}{2} x^T(t_f) S(t_f) x(t_f) + \frac{1}{2} \int_{t_0}^{t_f} \left(x^T Q x + u^T R u\right) dt$$

$$S(t_f) \ge 0, \quad Q \ge 0, \quad R > 0$$

$$-\dot{S}(t) = A^T S + SA - SBR^{-1}B^T S + Q, \quad t \le T, \quad \text{boundary condition } S(t_f)$$

$$K = R^{-1} B^T S$$

$$u = -Kx$$

$$J^*(t_0) = \frac{1}{2} x^T(t_0) S(t_0) x(t_0)$$