ELEC-E8116 Model-based control systems
Full exam. 5.4.2016

- *Write the name of the course, your name, your study program, and student number to each answer sheet.*
- *There are five (5) problems and each one must be answered.*
- *No literature is allowed. A function calculator can be used.*

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1. Explain briefly the following concepts

- Fundamental restrictions in control
- Singular value decomposition
- Waterbed effect
- Robust stability
- Small gain theorem

2.a. Draw a schema of the "one-degree-of-freedom" control configuration. Define the concepts *sensitivity function* and *complementary sensitivity function* for it.

2.b. Consider a SISO-case. Determine the region in the complex plane where $|S| > 1$. How can the result be explained in view of control performance?

3. Consider a MIMO system with the transfer function matrix

$$G(s) = \begin{bmatrix} \frac{2}{s+1} & \frac{3}{s+2} \\ \frac{1}{s+1} & \frac{1}{s+1} \end{bmatrix}$$

a. Determine the poles and zeros of the above system. What conclusions can be made with respect to control?

b. Calculate the *Relative Gain Array* RGA at zero frequency in the above example case. What conclusions can be made with respect to control?

4. Consider a nominal SISO plant $G(s)$ with an *additive* uncertainty Δ

$$G_0(s) = G(s) + \Delta(s), \quad \|\Delta(s)\|_\infty \le 1$$

Note that $G_0(s)$ is the true model and that the uncertainty is not multiplicative (like the one discussed in the lectures). The system is controlled by a 2 DOF controller with $F_r = 0$ and F_y.

Derive a condition for the closed-loop system to be robustly stable.

5. Consider the 1.order process $G(s) = \dfrac{1}{s-a}$, which has a realization

$$\dot{x}(t) = ax(t) + u(t)$$
$$y(t) = x(t)$$

so that the state is directly measurable . It is desired to find a control that minimizes the criterion

$$J = \int_0^\infty \left(x^2 + Ru^2\right) dt \qquad (R > 0)$$

Determine the solution and determine the closed loop state equation. Is the closed loop system stable, when the process is i. stable, ii. unstable?

Some formulas that might be useful:

$$\dot{x} = Ax + Bu, \quad t \ge t_0$$

$$J(t_0) = \frac{1}{2} x^T(t_f) S(t_f) x(t_f) + \frac{1}{2} \int_{t_0}^{t_f} \left(x^T Q x + u^T R u \right) dt$$

$$S(t_f) \ge 0, \quad Q \ge 0, \quad R > 0$$

$$-\dot{S}(t) = A^T S + SA - SBR^{-1}B^T S + Q, \quad t \le T, \quad \text{boundary condition } S(t_f)$$

$$K = R^{-1} B^T S$$

$$u = -Kx$$

$$J^*(t_0) = \frac{1}{2} x^T(t_0) S(t_0) x(t_0)$$