ELEC-E8116 Model-based control systems
Intermediate exam 2. 4. 4. 2017

- *Write the name of the course, your name, your study program, and student number to each answer sheet.*
- *There are three (3) problems and each one must be answered.*
- *No literature is allowed. A function calculator can be used.*
- *Mark clearly INTERMEDIATE EXAM 2 on your answer sheet*

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1. Consider a SISO system in a two-degrees-of-freedom control configuration. Let the loop transfer function be $L(j\omega) = G(j\omega)F_y(j\omega)$, where the symbols are standard used in the course.

a. Define the *sensitivity* and *complementary sensitivity functions* and determine where in the complex plane it holds

$$|S(j\omega)| < 1, \quad |S(j\omega)| = 1, \quad |T(j\omega)| < 1 \text{ and } |T(j\omega)| = 1$$

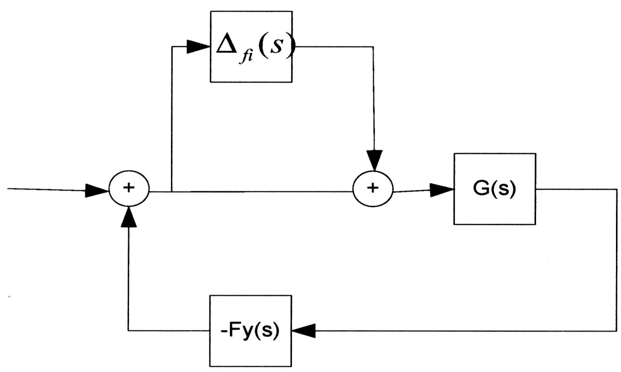
b. Determine the point(s) in the complex plane where $|S(j\omega)| = |T(j\omega)| = 1$.

Hint to the problem: In the complex plane (xy) let $L(j\omega) = x(\omega) + jy(\omega)$.

2. You are given the nominal plant

$$G(s) = \frac{10}{s^2 + 4}$$

with an input feedback uncertainty $\|\Delta_{fi}(s)\|_\infty \le 0.5$, and the controller $F_y(s) = \dfrac{4(s+2)}{s+8}$ (see Fig.). State the conditions for robust stability of the closed-loop system? (You do not have to perform the calculations to the end. Your result must however be such that an easy Matlab calculation would give the final result).



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3. Consider the process

$$\dot{x}(t) = au(t),\ x(0) = x_0 \qquad (a \text{ is a real constant})$$

and the cost criterion to be minimized

$$J = \int_0^\infty \left(x^2(t) + u^2(t)\right)dt$$

Determine the optimal control law, solution of the closed loop trajectory and the value of the cost criterion. Consider different values of a. Is the closed-loop system stable?

Some formulas that might be useful:

$$\dot{x} = Ax + Bu, \quad t \geq t_0$$

$$J(t_0) = \frac{1}{2}x^T(t_f)S(t_f)x(t_f) + \frac{1}{2}\int_{t_0}^{t_f}\left(x^T Qx + u^T Ru\right)dt$$

$$S(t_f) \geq 0, \quad Q \geq 0, \quad R > 0$$

$$-\dot{S}(t) = A^T S + SA - SBR^{-1}B^T S + Q, \quad t \leq T, \quad \text{boundary condition } S(t_f)$$

$$K = R^{-1}B^T S$$

$$u = -Kx$$

$$J^*(t_0) = \frac{1}{2}x^T(t_0)S(t_0)x(t_0)$$