ELEC-E8116 Model-based control systems

Full exam 4. 4. 2017

- *Write the name of the course, your name, your study program, and student number to each answer sheet.*
- *There are three (5) problems and each one must be answered.*
- *No literature is allowed. A function calculator can be used.*
- *Write clearly FULL EXAM on your answer sheet.*

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1. Explain briefly the following concepts

- Principle of Optimality
- Dynamic programming
- Waterbed effect
- Robust stability
- "Push-through" rule
- Internal model control

2. Consider a SISO system in a two-degrees-of-freedom control configuration. Let the loop transfer function be $L(j\omega) = G(j\omega)F_y(j\omega)$, where the symbols are standard used in the course.

a. Define the *sensitivity* and *complementary sensitivity functions* and determine where in the complex plane it holds

$$|S(j\omega)| < 1, \quad |S(j\omega| = 1, \quad |T(j\omega)| < 1 \text{ and } |T(j\omega)| = 1$$

b. Determine the point(s) in the complex plane where $|S(j\omega)| = |T(j\omega)| = 1$.

Hint to the problem: In the complex plane (xy) let $L(j\omega) = x(\omega) + jy(\omega)$.

3. a. Explain the *Small Gain Theorem*.

b. Explain shortly what is meant by the Relative Gain Array (RGA) and what is its meaning in control engineering.

c. Consider a linear SISO system. Explain shortly what different definitions there exist for the concept *bandwidth*. Explain these shortly. How can they be characterized in terms of control performance?

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4. Consider the system

$$\dot{x}_1(t) = -x_1(t) + u(t)$$
$$\dot{x}_2(t) = x_1(t)$$

The criterion to be minimized is

$$J = \int_0^\infty (x_2^2(t) + 0.1u^2(t))dt$$

Determine the optimal control law and the optimal cost.

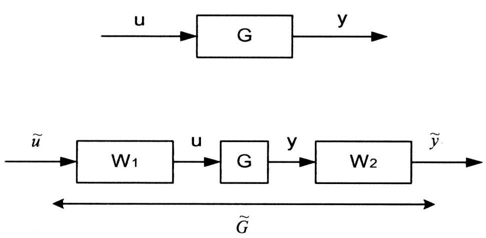
5. Consider the following multivariable system

$$G(s) = \begin{bmatrix} \frac{2}{s+1} & \frac{3}{s+2} \\ \frac{1}{s+1} & \frac{1}{s+1} \end{bmatrix}$$

for which at zero frequency

$$G(0) = \begin{bmatrix} 2 & 1.5 \\ 1 & 1 \end{bmatrix} \qquad \text{RGA}\left(G(0)\right) = \begin{bmatrix} 4 & -3 \\ -3 & 4 \end{bmatrix}$$

a. How would you choose the *pairing*, if two SISO loops would be used for control.
b. How would you design a *decoupling* multivariable controller, when the decoupling is designed at the zero frequency. You do not have to design the controller numerically, just explain in detail how you would do it.



Some formulas that might be useful:

$$\dot{x} = Ax + Bu, \quad t \geq t_0$$

$$J(t_0) = \frac{1}{2}x^T(t_f)S(t_f)x(t_f) + \frac{1}{2}\int_{t_0}^{t_f}\left(x^TQx + u^TRu\right)dt$$

$$S(t_f) \geq 0, \quad Q \geq 0, \quad R > 0$$

$$-\dot{S}(t) = A^TS + SA - SBR^{-1}B^TS + Q, \quad t \leq T, \quad \text{boundary condition } S(t_f)$$

$$K = R^{-1}B^TS$$

$$u = -Kx, \; J^*(t_0) = \frac{1}{2}x^T(t_0)S(t_0)x(t_0)$$