ELEC-C1320 – Robotiikka, Exam 15.5.2017 (3 hours)

<u>It is allowed</u> to use a calculator and a book of mathematical equations (e.g. MAOL) in the exam.

1. The rotation matrix ***R*** describes the orientation of a coordinate frame with respect to a world frame: $\boldsymbol{R} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix}$

   a) The x-axis of the coordinate frame is parallel to the: a) world frame x-axis, b) y-axis, c) z-axis or d) negative z-axis ? (2 points)

   b) The y-axis of the coordinate frame is parallel to the: a) world frame x-axis, b) y-axis, c) negative y-axis or d) z-axis ? (2 points)

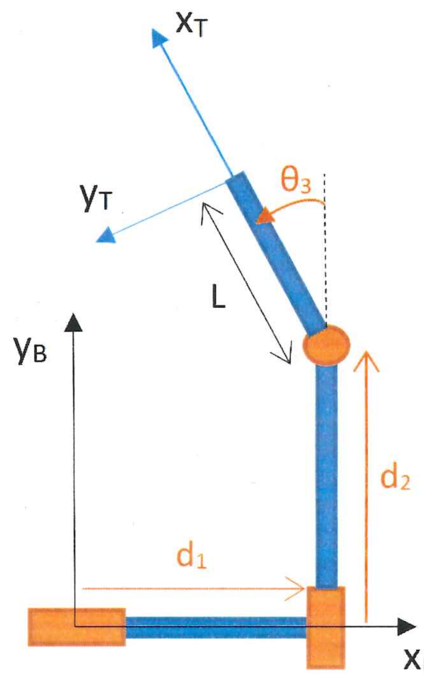
   c) The z-axis of the coordinate frame is parallel to the: a) world frame negative x-axis, b) negative y-axis, c) negative z-axis or d) z-axis ? (2 points)

   d) Does R describe a right-handed coordinate frame? Answer YES/NO (3 points)

2. In the figure a planar PPR robot is shown. The orientation of the last link when the joint variable θ_3 is zero is indicated with the dashed line. The direction of positive rotation of θ_3 is indicated with the arrow in the figure.

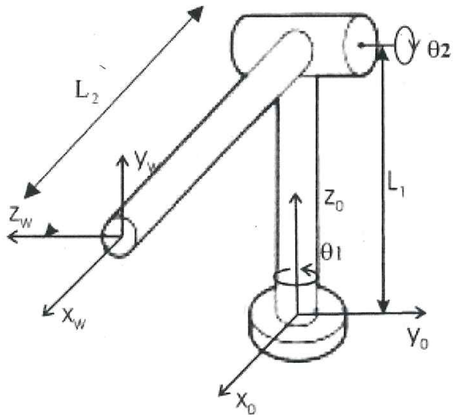
<u>Solve the forward kinematics problem of the manipulator to describe the tool frame (T) with respect to the robot base frame (B).</u> In other words, assign the link frames and provide the corresponding DenavitHartenberg-parameters in a table. It is your choice to use either the Standard or Modified DH-parameter convention. (18 points)

*Hint: You can assign the link frames into the given figure where all the joint variables have a **non-zero** value.*

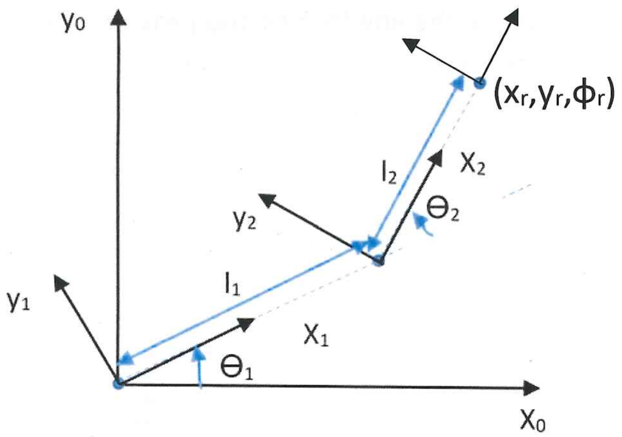


3. In the figure below a two degrees-of-freedom manipulator in its home/zero position is shown (upper arm oriented horizontally above the X_0-axis). Both degrees-of-freedom (dof) are rotational (the first rotating the upper link on the horizontal plane, Θ_1, and the second tilting the upper link with respect to the horizontal plane, Θ_2). Positive directions of rotations are also shown in the figure.

Find the inverse kinematic transform for the manipulator. (15 points)



4. Form the manipulator Jacobian matrix for the planar RR-manipulator shown in the figure below. (11 points)



The Jacobian matrix should obey the equation:

$$\begin{bmatrix} \dot{x}_r \\ \dot{y}_r \\ \dot{\phi}_r \end{bmatrix} = J_r(q) \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \end{bmatrix}$$

The forward kinematics model of the manipulator is:

$$x_r = l_1 c_1 + l_2 c_{12}$$
$$y_r = l_1 s_1 + l_2 s_{12}$$
$$\phi_r = \theta_1 + \theta_2$$

where $\mathbf{C}_{12}$ means $\cos(\theta_1 + \theta_2)$ and so on.

5. A weight of 3kg is fixed to the tip of last link (at the location of the origin of the H-frame) of the three-link manipulator shown in the figure below. The task is to calculate torques and forces affecting joints 1, 2 and 3 (due to gravity) in two different configurations of the manipulator arm. To solve the problem here **you must utilize the Jacobian matrix** of the manipulator. The joint configurations to be considered are:

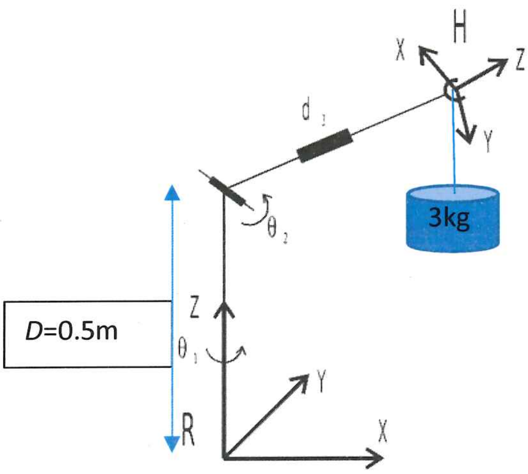
a) θ_1=90.0°, θ_2=30.0°, d_3=0.5m (*the total length of the upper link is described by d_3*) (6 points)

b) θ_1=90.0°, θ_2=90.0°,d_3=0.5m (*the total length of the upper link is described by d_3*) (6 points)

The links itself are assumed to be weightless.
The gravitational acceleration vector is pointing in the direction of negative Z_R-axis and its value is 9.81 m/s².

The Jacobian matrix of the manipulator is:

$$\begin{bmatrix} v_x \\ v_y \\ v_z \end{bmatrix} = J \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \\ \dot{d}_3 \end{bmatrix}, \qquad J = \begin{bmatrix} -s\theta_1 c\theta_2 d_3 & -c\theta_1 s\theta_2 d_3 & c\theta_1 c\theta_2 \\ c\theta_1 c\theta_2 d_3 & -s\theta_1 s\theta_2 d_3 & s\theta_1 c\theta_2 \\ 0 & c\theta_2 d_3 & s\theta_2 \end{bmatrix}$$



ELEC-C1320 Robotiikka - Equations

Link parameters and the corresponding elementary transformations as well as the symbolic form of the link matrix according to the **standard** Denavit and Hartenberg parameter convention:

$$^{j-1}A_j(\theta_j, d_j, a_j, \alpha_j) = T_{Rz}(\theta_j)T_z(d_j)T_x(a_j)T_{Rx}(\alpha_j)$$

$$^{j-1}A_j = \begin{pmatrix} \cos\theta_j & -\sin\theta_j\cos\alpha_j & \sin\theta_j\sin\alpha_j & a_j\cos\theta_j \\ \sin\theta_j & \cos\theta_j\cos\alpha_j & -\cos\theta_j\sin\alpha_j & a_j\sin\theta_i \\ 0 & \sin\alpha_j & \cos\alpha_j & d_j \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Link parameters and the corresponding elementary transformations as well as the symbolic form of the link matrix according to the **modified** Denavit and Hartenberg parameter convention:

$$^{j-1}A_j = R_x(\alpha_{j-1})T_x(a_{j-1})R_z(\theta_j)T_z(d_j)$$

$$^{j\text{-}1}\mathbf{A}_j = \begin{bmatrix} c\theta_j & -s\theta_j & 0 & a_{j-1} \\ s\theta_j c\alpha_{j-1} & c\theta_j c\alpha_{j-1} & -s\alpha_{j-1} & -s\alpha_{j-1}d_j \\ s\theta_j s\alpha_{j-1} & c\theta_j s\alpha_{j-1} & c\alpha_{j-1} & c\alpha_{j-1}d_j \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Elementary rotation transformations (i.e. rotations about principal axis by θ):

$$R_x(\theta) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{pmatrix}$$

$$R_y(\theta) = \begin{pmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{pmatrix}$$

$$R_z(\theta) = \begin{pmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Inverse of a 4x4 transformation matrix:

$$T^{-1} = \begin{pmatrix} R & t \\ 0_{1\times 3} & 1 \end{pmatrix}^{-1} = \begin{pmatrix} R^T & -R^T t \\ 0_{1\times 3} & 1 \end{pmatrix} \qquad (2.21)$$

Derivation of trigonometric functions:

Dsinx = cosx

Dcosx = -sinx

Definition of (manipulator) Jacobian matrix:

If $\boldsymbol{y} = F(\boldsymbol{x})$ and $\boldsymbol{x} \in \mathbb{R}^n$ and $\boldsymbol{y} \in \mathbb{R}^m$ then the Jacobian is the $m \times n$ matrix

$$J = \frac{\partial F}{\partial \boldsymbol{x}} = \begin{pmatrix} \frac{\partial y_1}{\partial x_1} & \cdots & \frac{\partial y_1}{\partial x_n} \\ \vdots & \ddots & \vdots \\ \frac{\partial y_m}{\partial x_1} & \cdots & \frac{\partial y_m}{\partial x_n} \end{pmatrix}$$

Jacobian transpose transforms a wrench applied at the end-effector, ${}^0\boldsymbol{g}$ to torques and forces experienced at the joints $\boldsymbol{Q}$:

$$\boldsymbol{Q} = {}^0J(q)^T \, {}^0\boldsymbol{g}$$