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CS-E3200 – Discrete Models and Search (5 cr)
Exam May 24, 2017

Write down on each answer sheet:
- Your name, degree program, and student number.
- The text: "CS-E3200 Discrete Models and Search 24.5.2017".
- The total number of answer sheets you are submitting for grading.

You can write down your answers in either Finnish, Swedish, or English. Calculators are allowed.

1. Propositional logic: *(total: 10 points)*
   (a) Let φ be the formula $(p \to q \vee s) \wedge (r \to s) \to (((p \to q) \to r) \to s)$. Convert φ into DNF. *(3 points)*
   (b) Show that the formula φ is *valid* using a truth table. *(3 points)*
   (c) Moreover, prove that φ is valid using refutation by resolution. *(4 points)*

2. Consider the 9×9 Sudoku puzzle (as illustrated in Figure 1 below) where a partially filled 9×9 table is given, and you are asked to fill each blank cell with a number in $1, 2, \ldots, 9$ such that the set of numbers in each row, each column and each of the nine 3×3 small squares is equal to the set $\{1, \ldots, 9\}$. Now, design a constraint satisfaction problem which models a given Sudoku puzzle instance. Describe your variables *(2 points)*, the domain of each variable *(2 points)*, and the constraints over those variables *(3 points)*. Now, extend your CSP model of Sudoku to any $n^2 \times n^2$ instance of Sudoku puzzle *(3 points)*. *(total: 10 points)*

3. Use the initial assignment $\tau = \{a \mapsto 1, b \mapsto 0, c \mapsto 1, d \mapsto 0, e \mapsto 1\}$ to simulate 5 steps of the GSAT algorithm on the following clauses *(2 points/step)*. Describe the reasons for changes from step i to $i+1$. *(total: 10 points)*

   $c_1 : \neg a \vee \neg b$ $\quad c_2 : \neg a \vee b$ $\quad c_3 : a \vee \neg e$ $\quad c_4 : \neg a \vee d$ $\quad c_5 : d \vee e$
   $c_6 : b \vee \neg c$ $\quad c_7 : b \vee \neg e$ $\quad c_8 : b \vee \neg c \vee \neg d$ $\quad c_9 : \neg c \vee \neg d$ $\quad c_{10} : \neg c \vee \neg e$

4. Assume that x and y are integers in the range $[-10, 10]$ and $b_1, b_2, \ldots, b_5$ are binary variables. Do the followings using integer linear programming: *(total: 10 points)*
   (i) Express $b_1 = 1$ if and only if $x \geq 3$. *(2 points)*
   (ii) Express $b_2 = 1$ if and only if $x \leq -3$. *(2 points)*
   (iii) Use b_3, b_4 and b_5 to express $b_3 = 1$ if and only if $y = 2$. *(2 points)*
   (iv) Use b_1, b_2 and b_3 as above to express "$|x| \geq 3$ if and only if $y = 2$". *(2 points)*
   (v) Obtain concrete values for big M's that you have used above. *(2 points)*

5. Consider the following integer linear program:

   $$\max\ 3x_1 + 6x_2 \quad \text{such that} \quad x_1 - x_2 \leq 1, \qquad x_1 - 4x_2 \geq -8, \qquad x_1, x_2 \geq 0, \qquad x_1, x_2 \text{ are integers.}$$

   Give its linear relaxation *(1 point)*. Transform the linear relaxation to standard form *(2 points)*. Write this relaxed problem as a Simplex tableau with a basis that corresponds to when $x_1 = x_2 = 0$ *(2 points)*. Given such a starting point, simulate Simplex algorithm to obtain the optimal solution to the relaxed problem *(5 points)*. *(total: 10 points)*

			4	6				3
			3			4	1	
	1	4						
8		1	7		2		3	
	2		8		3	6		9
						9	2	
	8	3			6			
5				4	9			

Figure 1: A sample instance of a 9×9 Sudoku puzzle. It consists of nine 3×3 squares that, together, form a 9×9 square. Some cells are pre-filled with numbers 1 to 9. A solution should fill blank cells with numbers 1 to 9 so that the set of numbers in each row, each column, and each small 3×3 square is equal to the set $\{1, \ldots, 9\}$.