Aalto University School of Science, Department of Information and Computer Science JR

T-79.3001 Logic in Computer Science: Foundations

Examination, December 16, 2013

Please note the following: your answers will be graded only if you have completed the three obligatory home assignments before the exam!

Assignment 1 $(1+1+8p)$
Two 3-bit binary numbers in the range 0 to 7 are represented by the sequences of propositional variables $x_2x_1x_0$ and $y_2y_1y_0$, where x_2 and y_2 are respectively the most significant bits of the numbers.

1. Construct a propositional formula that is true if and only if $x_2x_1x_0$ is obtained from $y_2y_1y_0$ by doing a rotation one bit to the right.
2. Construct a propositional formula which is true if and only if the value of the binary number $x_2x_1x_0$ is 6.
3. Construct a propositional formula that is true if and only if the relation $x_2x_1x_0 \leq y_2y_1y_0$ holds, that is, the first number is less than or equal to the second number.

Assignment 2 $(2+3+5p)$ Let ϕ_1, ϕ_2, and ϕ_3 be sentences of the propositional logic. Justify your answer in each case.

(a) If ϕ_1 is satisfiable and ϕ_2 is valid, is $\phi_1 \land \phi_2$ satisfiable? Is it valid?

(b) Assume that $\phi_1 \models \phi_2$ and $\neg(\neg\phi_2 \lor \phi_3)$ is not satisfiable. What can you say about the relation between ϕ_1 and ϕ_3?

(c) Let ϕ be any sentence in the propositional logic that consists of atomic propositions as well as connectives $\lor$, $\land$ and $\neg$, with at most one occurrence of each atomic proposition in ϕ. Claim: ϕ is satisfiable. Is this claim necessarily true? Give a proof sketch or give a counter-example.

Assignment 3 $(10p)$ Prove the following claims using semantic tableaux:

(a) $\models (A \to B) \to ((C \to D) \to (A \land C \to B \land D))$.

(b) $\models \neg(\exists x \forall y \exists z \neg P(x,y,z) \land \forall z \exists y \forall x P(z,y,x))$.

Tableau proofs must contain all intermediate steps !!!

Assignment 4 $(10p)$ Derive a Prenex normal form and a clausal form (i.e., a set of clauses S) for the sentence $\neg(\exists x(A(x) \lor B(x) \lor C(x)) \to \exists x A(x) \lor \exists y B(y) \lor \exists z C(z))$.

Make S as simple as possible. Prove that S is unsatisfiable using resolution.

Assignment 5 $(10p)$

(a) Derive for the program `if(x<y) then{z=y-x} else{z=x-y}` the *weakest precondition* strarting from the *postcondition* `(z>0)`. $(4p)$

(b) Consider the following program `Triple`:

```
v=x; z=x; while(!(z==0)) {z=z-1; v=v+2}.
```

Use weakest preconditions and a suitable invariant $(6p)$ to establish

$$\models_p [\texttt{true}]\ \texttt{Triple}\ [\texttt{v==3*x}].$$