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T-79.1001 Introduction to Theoretical Computer Science T (4 cr)
Exam Friday February 21st, 2014, 9:00–12:00

Ensure that every answer sheet contains:
- Your name, degree programme, student number
- Course name "T-79.1001 Introduction to Theoretical Computer Science T" and the date "Feb 21, 2014"
- The total number of answer sheets submitted for grading

Use of calculators is not allowed in the exam.
Note: if you have not completed your computerized home assignments, your exam will not be graded.

1. (a) Give a deterministic finite state automaton for the language
   $L = \{w \in \{a,b\}^* \mid |w| \text{ is odd and } w \text{ has an even number of } a\text{'s}\}$ *5p.*
   (b) Give a regular expression generating the language
   $L = \{w \in \{a,b\}^* \mid w \text{ contains at least two } a\text{'s and at most one } b\}$ *5p.*
   (c) Give a regular expression generating the language
   $L = \{w \in \{a,b\}^* \mid w \text{ does not contain the substring } abb\}$
   (Hint: you may want to first design a deterministic automaton for L) *5p.*

2. Consider the language

$$L = \{a^{2k}b^n b^k \mid k \geq 0 \text{ and } n \geq 0\}$$

   over the alphabet $\{a,b\}$.
   (a) Show that L is not regular. *6p.*
   (b) Design a context free grammar that produces L. *5p.*
   (c) Give parse trees for the strings aab and $aabb$ in your grammar. *2p.*
   (d) Is your grammar in Chomsky normal form? If not, give one normal form requirement that is violated in your grammar. *2p.*

3. Design a pushdown automaton that decides whether the input belongs to the language

$$L = \{xby \mid x,y \in \{a,b\}^* \text{ and } |x| = |y|\}$$

   Is your automaton deterministic or nondeterministic? Present computations of your automaton with inputs abb, aab, and $abaa$. *15p.*

Continued on the other side

4. (a) Define the notions of a recursive ("decidable") and recursively enumerable ("semidecidable") language.
Is the language $L_{\text{primeprod}} = \{x \in \{0,1,...,9\}^* \mid x \text{ is a product of two prime numbers}\}$ recursive? Justify your answer. (E.g. 15 belongs to the language as $15 = 3 \times 5$ but 16 is not in the language.) *5p.*

(b) Given a language L over an alphabet Σ, let $L^R = \{w^R \mid w \in L\}$ be the language obtained by reversing each string in L. Here w^R is the reverse of w (for example, $(gnat)^R = tang$). Show the following claim either correct or incorrect: if L is a recursive language, then L^R is also a recursive language. *5p.*

(c) Prove that the following problem is undecidable: given an arbitrary Turing machine, is the language accepted by it finite? That is, prove that the language $L_{\text{fin}} = \{c \in \{0,1\}^* \mid L(M_c) \text{ is finite}\}$ is not recursive, where M_c denotes the Turing machine encoded by the string c. If you use "Rice's theorem" (you don't have to), give a precice definition for it as well. *5p.*

Total 60p.